

The Quantum-GR Gluing Obstruction: Admissibility Algebra, Policy-Indexed Identity, and the Sheaf-Theoretic Structure of Quantum Gravity

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Quantum gravity is often described as the unfinished unification of quantum theory and general relativity, but this framing may hide the deeper obstruction. The two theories do not merely assign different equations to reality; they assign different admissibility conditions for when descriptions, states, geometries, observables, and transformations count as physically identical. Quantum theory treats identity through Hilbert-space structure, operator algebras, measurement context, phase, entanglement, and unitarity, while general relativity treats identity through diffeomorphism invariance, causal structure, metric geometry, covariance, and background independence. This paper proposes that the quantum-GR problem is therefore a gluing problem, not only a quantization problem. Using Admissibility Algebra, category theory, sheaf-theoretic gluing, and homotopy type-theoretic identity, we formulate quantum and gravitational descriptions as policy-indexed quotient presentations of a deeper physical object. Classical spacetime appears when quotient presentations stabilize across broad admissibility families; quantum ambiguity appears when no global policy-independent section exists. The central thesis is that quantum theory and general relativity may glue only after identity itself becomes admissibility-covariant, replacing forced unification with lawful compatibility across contexts, scales, and observers.

Keywords: Admissibility Algebra, quantum gravity, general relativity, quantum theory, mathematical physics, gluing obstruction, sheaf theory, category theory, homotopy type theory, policy-indexed identity, diffeomorphism invariance, gauge symmetry, measurement context, quantum measurement, spacetime emergence, background independence, algebraic quantum field theory, quantum field theory in curved spacetime, holography, decoherence, contextuality, univalence, quotient physics, admissibility-covariant physics.

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1. The Quantum-GR Problem as an Identity Problem

The usual description of quantum gravity begins with a failure of unification. Quantum theory governs microscopic systems, probability amplitudes, superposition, entanglement, measurement, and operator-valued observables. General relativity governs spacetime geometry, causal structure, gravitation, covariance, and the dynamical relation between matter and metric. The two theories are both extraordinarily successful within their native regimes, yet they resist merger at the domains where both must apply: black holes, the early universe, singularities, Planck-scale structure, and any situation in which quantum matter and spacetime geometry can no longer be cleanly separated. This paper argues that the deepest obstruction is not simply that one theory is quantum and the other geometric. The deeper obstruction is that the two theories do not use the same rules for determining when two descriptions count as the same physical object.

This difference is usually treated as a technical difficulty. One attempts to quantize gravity, geometrize quantum theory, sum over histories, replace spacetime by networks, encode gravity holographically, or derive spacetime from entanglement. These programs may each capture important portions of the problem. But beneath them lies a prior question: what is the admissible identity relation in the theory being constructed? In quantum theory, identity is tied to state vectors up to phase, density operators, operator algebras, measurement contexts, unitary equivalence, commutation structure, and probability assignments. In general relativity, identity is tied to diffeomorphism invariance, tensorial covariance, causal structure, metric relations, gauge redundancy, and coordinate-independent geometry. These are not merely different variables. They are different policies for sameness.

Admissibility Algebra provides language for this hidden layer. A physical theory does not merely specify objects and equations. It specifies which transformations preserve the identity that matters. It defines what may be substituted for what without changing the physical content. In that sense, every mature physical theory already contains an admissibility policy, even when that policy remains implicit. Quantum theory has one family of admissibility policies; general relativity has another. The quantum-GR problem is therefore not only a problem of dynamics, regularization, renormalization, or ontology. It is a problem of gluing incompatible identity policies into a larger structure without erasing the valid invariants of either theory.

1.1 Unification versus gluing

Unification seeks a single master theory from which quantum theory and general relativity appear as limiting cases. This is a natural ambition. Physics has repeatedly advanced by unifying domains once considered separate: terrestrial and celestial mechanics, electricity and magnetism, space and time, matter and radiation, weak and electromagnetic interactions. The

prestige of unification is therefore justified. Yet quantum gravity may require a different conceptual operation. The relevant task may not be to force quantum theory and general relativity into one common presentation, but to identify a higher structure in which their distinct presentations become compatible local charts.

A chart is not the territory. A coordinate system is not spacetime. A basis is not a vector space. A gauge choice is not the field itself. A measurement arrangement is not the quantum object independent of all arrangements. In each case, physics survives by distinguishing presentation from invariant content. The problem with quantum gravity is that quantum theory and general relativity appear to disagree about how that distinction should be drawn. One theory localizes identity in algebraic and measurement-theoretic structure; the other localizes it in geometric and diffeomorphic structure.

Gluing is the more appropriate metaphor when different local descriptions are valid but cannot be globally identified by naive equality. In geometry, local coordinate patches can describe one manifold even though no single coordinate system covers the whole object without singularities or distortion. In sheaf theory, local data may or may not assemble into a global section. In gauge theory, local potentials can describe a global field only when transition functions obey compatibility conditions. In quantum theory, different measurement contexts may generate locally meaningful outcomes without admitting a single context-independent assignment of values. These are not failures of reality. They are failures of overly simple identity.

The proposal of this paper is that quantum theory and general relativity should be treated as local presentations over different regions of a larger admissibility space. Where their policies agree, gluing is possible. Where their policies conflict, obstruction appears. Quantum gravity is then not merely the name of a missing equation. It is the name of the missing gluing law between quantum admissibility and geometric admissibility.

1.2 Why quantum theory and general relativity disagree about sameness

Quantum theory and general relativity both reject naive object identity, but they reject it in different ways. Quantum theory rejects the idea that a physical system is fully described by a list of pre-existing classical properties. A quantum state encodes amplitudes, probabilities, correlations, and possible outcomes relative to observables. Two state vectors differing only by a global phase may represent the same physical state. Mixed states may encode indistinguishability at the level of accessible statistics. Entangled systems may lack separable identity for their parts even when the composite state is well defined. Noncommuting observables resist simultaneous value assignment. Measurement changes the admissible description.

General relativity rejects naive identity through covariance and diffeomorphism invariance. Coordinates have no physical authority by themselves. A point of the manifold has no intrinsic physical identity independent of the fields and relations in which it participates. Two metric-field configurations related by a diffeomorphism are not two distinct physical universes in the ordinary sense; they are two presentations of the same geometry. What matters is not the coordinate label but the invariant relational structure: causal cones, curvature, geodesics, proper time, and the dynamical relation between matter and metric.

Both theories therefore teach that reality is not identical with its presentation. Yet their admissibility rules differ. In quantum mechanics, admissible identity is often organized around Hilbert-space rays, operators, expectation values, spectra, transition amplitudes, and contextual measurement outcomes. In general relativity, admissible identity is organized around diffeomorphism classes, tensorial relations, causal order, and background-independent geometry. The mismatch becomes acute when one asks what it means for a quantum state of geometry to be the same as another, or what it means for a quantum measurement to occur when the temporal and spatial background needed to define the measurement is itself dynamical.

This disagreement about sameness is not cosmetic. It affects what counts as an observable, what counts as evolution, what counts as locality, what counts as information, and what counts as the system under study. A quantum field normally presupposes a spacetime background sufficient to define localization and time evolution. General relativity says that the background is not fixed but dynamical. The ordinary quantum question asks how a state changes in time. The gravitational question asks how time itself emerges from a dynamical geometry. These are not merely different equations placed on the same stage. They are different rules for constructing the stage.

1.3 Physical identity as an admissibility condition

Admissibility Algebra begins with a simple shift. Instead of asking whether two objects are equal in an absolute sense, one asks under what policy they may be treated as identical for a given purpose. A policy determines the invariants that must be preserved, the transformations that are allowed, the distinctions that may be discarded, and the quotient object that results. In mathematical language, a presentation X becomes a quotient $[X]\pi$ under an admissibility policy π . The quotient does not deny the presentation. It identifies the content that survives the policy.

Physical theories are filled with such quotient operations. Coordinate systems are quotiented by covariance. Gauge potentials are quotiented by gauge equivalence. Quantum state vectors are quotiented by physically irrelevant phase. Thermodynamic systems are quotiented by macroscopic variables that ignore microscopic microstate distinctions. Classical objects are

quotiented by enormous environmental averaging and stability conditions. Effective field theories quotient away degrees of freedom irrelevant at a chosen scale. Every successful theory therefore operates not only with objects, but with admissible forgettings.

The crucial issue is that different policies preserve different invariants. A policy suited for quantum interference may preserve phase relations that a classical coarse-graining discards. A policy suited for general covariance may discard coordinate distinctions that a laboratory measurement protocol temporarily uses. A policy suited for local quantum field theory may require a background causal structure that a full theory of gravity cannot assume as fixed. A policy suited for black hole thermodynamics may preserve entropy and horizon structure while suppressing microscopic distinctions. No single naive equality relation can serve all these roles.

Physical identity is therefore not a primitive absolute. It is a disciplined equivalence relation under a policy. To say that two descriptions are physically the same is to say that the differences between them are inadmissible as physical differences under the policy in force. This does not make physics subjective. It makes physics explicitly structured. A good admissibility policy is not arbitrary; it is constrained by experiment, mathematical coherence, invariance, transformation law, reproducibility, and compatibility with neighboring policies. The point is not that anything may be treated as anything else. The point is that lawful substitution requires an explicit rule.

In quantum gravity, the missing rule is the policy under which quantum identity and gravitational identity become jointly admissible. Without such a policy, one risks preserving too much, preserving too little, or preserving mutually incompatible structures. Preserve the quantum formalism too rigidly, and background independence becomes difficult. Preserve geometric covariance too rigidly, and conventional quantum time evolution becomes difficult. The gluing problem is to identify the admissibility conditions under which both theories can be seen as quotient presentations of a deeper structure.

1.4 From equation conflict to quotient conflict

The visible conflict between quantum theory and general relativity often appears as an equation conflict. One equation describes unitary evolution or quantum amplitudes; another describes curvature sourced by stress-energy. One formalism works in Hilbert space; the other works on differentiable manifolds with Lorentzian metrics. One uses operators; the other uses tensors. One is probabilistic; the other is geometric. These differences matter, but they are not yet the root.

The root is quotient conflict. Quantum theory quotients the world in one way; general relativity quotients it in another. In quantum theory, many distinctions between state-vector presentations are physically irrelevant, while distinctions between noncommuting observables are deeply relevant. In general relativity, many distinctions between coordinate presentations

are physically irrelevant, while distinctions in causal and curvature structure are deeply relevant. The two theories carve the space of possible descriptions by different equivalence relations.

When two quotient systems are compatible, one can pass between them without contradiction. For example, classical mechanics can emerge from quantum mechanics under suitable limits, approximations, and decoherence conditions. Thermodynamics can emerge from statistical mechanics through coarse-graining and stability of macroscopic variables. Gauge field presentations can be patched through transition functions. These are cases where quotient structures can be related, even if they are not identical at the presentation level.

The quantum-GR case is harder because the quotienting operations interfere with each other. Quantum theory often needs a time parameter, a Hilbert-space decomposition, and an observable algebra defined with respect to a causal structure. General relativity makes causal structure dynamical. Quantum localization assumes enough geometry to say where an event or field excitation is. General relativity says geometry is part of what must be solved. Quantum measurement assumes a distinction between system, apparatus, and environment. In a cosmological or gravitationally closed system, that distinction becomes nontrivial. The ordinary quotient used by quantum theory presupposes structures that the gravitational quotient refuses to treat as fixed.

This suggests that quantum gravity should be formulated as a higher quotient problem. The deeper object is not simply a quantum state on spacetime, nor a classical spacetime containing quantum matter. It is a structure whose different quotient presentations yield quantum behavior under one family of policies and geometric behavior under another. The obstruction appears when one attempts to choose a single global presentation that preserves both identity systems without modification.

1.5 Statement of the central thesis

The central thesis of this paper is that the quantum-GR problem is an admissibility gluing obstruction. Quantum theory and general relativity do not merely disagree about what exists. They disagree about the admissibility policy under which existence, identity, observability, locality, and evolution are defined. A successful theory of quantum gravity must therefore provide not only new equations, but a higher identity discipline capable of explaining when quantum and geometric descriptions are compatible, when they fail to globalize, and what quotient object is preserved across their overlap.

In this framework, a physical object is not an absolute presentation X . It is a family of admissible quotient presentations $[X]_{\pi}$ indexed by policies π . Some policies are measurement-theoretic. Some are gauge-theoretic. Some are diffeomorphic. Some are scale-dependent. Some are observer-relative. Some are environmental. Some are algebraic. Some are geometric. The task

of mathematical physics is to determine which quotient presentations glue, which fail to glue, and what invariants survive the transition.

Classical spacetime is then interpreted as a stable quotient family: a regime in which many admissibility policies agree sufficiently to produce a robust geometric object. Quantum ambiguity is interpreted as failure of policy-independent globalization: a regime in which admissible descriptions remain contextual, noncommuting, entangled, or obstructed. Gravity is interpreted as the geometry of admissible relational structure. Quantum theory is interpreted as the algebra of admissible contextual structure. Quantum gravity is the sought higher structure in which these two become compatible without either being falsely reduced to the other.

The slogan is therefore precise: quantum theory and general relativity do not primarily fail to unify because their equations are written in different languages; they fail because their identity policies do not yet glue. The solution, if it exists, will not be a mere translation dictionary between operators and metrics. It will be an admissibility-covariant theory of physical identity.

2. Admissibility Algebra and Policy-Indexed Reality

Admissibility Algebra begins from the observation that no mathematical or physical theory operates with raw objects alone. Every theory also carries rules for when two presentations may be treated as the same, which distinctions must be preserved, which distinctions may be discarded, and which transformations are lawful. These rules are often invisible because they are built into the formalism. In group theory, admissible identity may be controlled by isomorphism. In topology, it may be controlled by homeomorphism or homotopy equivalence. In gauge theory, it may be controlled by gauge transformation. In general relativity, it may be controlled by diffeomorphism invariance. In quantum theory, it may be controlled by phase equivalence, unitary equivalence, operator algebra, and measurement context. Admissibility Algebra makes this hidden layer explicit.

The basic claim is that identity is not exhausted by equality. Equality is a strict relation inside a chosen presentation. Physical identity is usually weaker, richer, and policy-dependent. Two coordinate descriptions may be unequal as functions and yet represent the same spacetime geometry. Two vector representatives may be unequal as vectors and yet represent the same quantum ray. Two microscopic states may be unequal in detail and yet represent the same thermodynamic macrostate. Two field potentials may be unequal and yet yield the same field strength. These are not errors or approximations. They are lawful quotient operations.

A presentation X becomes physically or mathematically meaningful only after an admissibility policy π determines which transformations preserve the relevant content. The resulting object is

not simply X , but $[X]\pi$, the quotient of X under π . The policy π does not merely label an observer's opinion. It specifies the invariants, tests, transformations, and equivalence rules by which X can be used without destroying the identity that matters in context. A theory becomes mature when it can state not only its objects, but its admissibility discipline.

In mathematical physics, this shift is especially important because theories often fail to combine when their identity policies disagree. Quantum theory and general relativity do not merely use different mathematical machinery. They preserve different invariants and discard different distinctions. Admissibility Algebra supplies a way to compare such theories without prematurely forcing them into one language. It treats them as policy-indexed quotient systems and asks whether their quotient presentations can be related, transported, restricted, refined, or glued.

2.1 Presentations, policies, quotients, and invariants

A presentation is the form in which an object is given before admissibility has been applied. It may be a coordinate expression, a state vector, a metric tensor, a field configuration, a diagram, an algebra, a graph, a database entry, a proof term, or any structured object capable of transformation. A presentation is not necessarily false or superficial. It is simply not yet the final physical object, because it may contain distinctions that do not matter under the relevant policy.

An admissibility policy π is a rule that determines which differences in a presentation are allowed to be ignored and which must be preserved. It selects a class of transformations, a family of invariants, a criterion of substitution, and a quotient structure. The quotient $[X]\pi$ is the object X as seen through policy π . The map $q\pi: X \rightarrow [X]\pi$ sends a raw or detailed presentation to its admissible form. In practice, $q\pi$ may identify gauge-equivalent fields, phase-equivalent quantum states, diffeomorphic geometries, thermodynamically equivalent microstates, or observationally indistinguishable configurations.

The invariant $\text{Inv}\pi(X)$ is the content of X that survives policy π . It is what may be transported, compared, predicted, or measured without violating admissibility. The invariant is not always a number. It may be an algebra, a spectrum, a causal order, a curvature class, a probability distribution, a homotopy type, a conserved quantity, a symmetry class, or a relation among observables. The invariant is the stable residue of the quotient.

This gives the elementary pattern:

presentation $X \rightarrow$ admissibility policy $\pi \rightarrow$ quotient $[X]\pi \rightarrow$ invariant $\text{Inv}\pi(X)$

The point is not that the presentation is meaningless. Rather, the presentation is meaningful only relative to the admissibility policy that tells us what role it plays. A coordinate chart is useful, but only because covariance tells us which coordinate artifacts are not physical. A wavefunction is useful, but only because quantum theory tells us how phase, measurement,

and probability are to be interpreted. A metric expression is useful, but only because general relativity tells us how diffeomorphic presentations are to be compared.

In this framework, a physical theory is not merely a list of equations. It is a structured pair consisting of presentations and admissibility rules. The equations describe lawful relations among presentations. The admissibility policy determines which presentations count as physically equivalent. The quotient object is what the theory regards as real for the purpose at hand.

2.2 The admissibility policy π

The symbol π denotes a policy of admissible identity. It is not a single universal object. It may encode gauge convention, measurement context, observational resolution, dynamical regime, scale, causal accessibility, algebraic presentation, geometric background, coarse-graining rule, or observer frame. In ordinary physics these are usually treated separately. Admissibility Algebra treats them as members of a common family: rules governing lawful substitution.

A policy π must answer several questions. What transformations are allowed? What invariants must remain fixed? What distinctions are regarded as physically meaningful? What distinctions are regarded as representational? What observations or operations can detect a difference? What class of maps preserves identity? What quotient is produced? What neighboring policies can π communicate with?

For example, in gauge theory, π may say that potentials differing by an allowed gauge transformation represent the same physical field. In general relativity, π may say that diffeomorphic metric-field configurations are equivalent. In quantum theory, π may say that global phase is not physically observable, while relative phase may be. In thermodynamics, π may say that microstates with the same macrovariables are equivalent for the macroscopic theory. In effective field theory, π may say that high-energy degrees of freedom are integrated out at the chosen scale.

The same presentation X may produce different quotient objects under different policies. Thus $[X]\pi$ and $[X]\rho$ need not agree. This is not a defect. It is the central feature. A molecule may be treated as a quantum object under one policy, a classical object under another, a thermodynamic participant under another, and a biochemical functional unit under another. The molecule is not changing arbitrarily. The admissibility conditions are changing, and therefore the identity preserved by the theory changes.

A policy is not subjective whim. It is admissible only if it is lawful, coherent, and constrained. It must preserve the invariants it claims to preserve. It must not identify objects whose differences are detectable within the regime it governs. It must produce stable predictions when used within its domain. It must interact coherently with neighboring policies. A bad policy

collapses real distinctions. An overstrict policy preserves irrelevant distinctions and prevents useful theory formation. A mature theory chooses policies that are neither too coarse nor too fine.

The quantum-GR problem becomes clearer in this language. Quantum theory and general relativity use powerful admissibility policies, but those policies are not obviously compatible. Quantum policies often require external time, observable algebras, state spaces, and measurement contexts. Gravitational policies often require background independence, diffeomorphism invariance, dynamical causal structure, and geometric covariance. The question is not merely how to quantize a metric. The question is what policy π can lawfully preserve both quantum and gravitational identity.

2.3 Physical objects as quotient presentations

A physical object is often mistaken for a presentation. In classical intuition, a particle, field, or spacetime point appears to be simply there. Mathematical physics has repeatedly shown that this intuition is too rigid. What is physically real is usually an equivalence class, an invariant structure, or a quotient presentation. Admissibility Algebra states this directly: the physical object corresponding to X under policy π is $[X]\pi$.

This is not a denial of reality. It is a refinement of realism. The object is real not because every feature of its presentation is absolute, but because a stable invariant survives admissible transformation. The reality of an electromagnetic field does not depend on one chosen potential. The reality of a spacetime geometry does not depend on one coordinate chart. The reality of a quantum state does not depend on multiplying a state vector by an unobservable global phase. The reality of temperature does not depend on enumerating every molecular microstate.

The quotient form $[X]\pi$ captures the object as a lawful identity class. Each member of the class is a presentation. The quotient is the identity preserved across the admissible class. If X and Y are presentations, then $X \equiv_{\pi} Y$ means that X and Y are identical under policy π . This does not imply $X = Y$ as raw presentations. It means that all π -relevant invariants agree.

This distinction prevents a common error in foundational physics: treating failure of presentation identity as failure of physical identity. If two gauges differ, one has not necessarily found two physical fields. If two coordinate descriptions differ, one has not necessarily found two spacetimes. If two quantum representatives differ by irrelevant phase, one has not necessarily found two quantum states. Physical identity lies at the quotient level.

However, the reverse error is also dangerous. One must not quotient away distinctions that matter. Relative phase may be physically crucial. Curvature may be physically crucial. Entanglement structure may be physically crucial. Causal order may be physically crucial. A

policy is admissible only when it preserves the distinctions required by the theory's empirical and mathematical commitments.

A physical object should therefore be understood as a quotient presentation supported by invariants and disciplined by admissibility. This applies across scales. A black hole may be represented by a classical metric, thermodynamic variables, quantum microstates, horizon algebra, holographic boundary data, or information-theoretic constraints. Each is a different quotient presentation. The problem is not that only one can be real. The problem is to determine how these quotient presentations relate and whether they glue into a coherent higher object.

2.4 Compatibility, incompatibility, and admissible transition

Once physical objects are treated as policy-indexed quotients, the next question is how different policies relate. Given two policies π and ρ , there may or may not be an admissible transition from $[X]\pi$ to $[X]\rho$. Such a transition exists when the information preserved by one quotient can be lawfully transformed, restricted, refined, or compared with the information preserved by the other.

Compatibility does not require identity of policies. Classical mechanics and thermodynamics are not the same policy, but they can be related through statistical mechanics and limiting procedures. Quantum mechanics and classical mechanics are not the same policy, but decoherence, coarse-graining, and semiclassical limits can relate them. Gauge potentials and field strengths are not the same presentation, but gauge quotienting produces stable field observables. Local coordinate charts and global manifolds are not the same object, but transition maps allow coherent gluing.

Incompatibility occurs when two policies preserve structures that cannot be simultaneously globalized. In quantum theory, incompatible observables resist simultaneous sharp value assignment. In sheaf language, local sections may fail to glue into a global section. In gauge theory, an obstruction may prevent trivialization over a whole space. In quantum gravity, the incompatibility may involve time, locality, observability, and identity. A policy that requires fixed causal background may not be compatible with a policy that makes causal background dynamical.

An admissible transition $T_{\pi\rho}: [X]\pi \rightarrow [X]\rho$ must preserve the content that both policies are entitled to compare. Such a transition may be exact, approximate, asymptotic, coarse-grained, probabilistic, functorial, or obstructed. Exact transitions are rare in deep physics. More often, one obtains partial compatibility: a classical limit, a semiclassical approximation, an effective field theory, a local construction, or a duality valid under special conditions.

A transition becomes gluing data when it connects local or policy-indexed descriptions into a broader structure. If $[X]\pi$, $[X]\rho$, and $[X]\sigma$ are quotient presentations under different policies, then gluing requires not only pairwise transition maps, but compatibility among transitions. Moving from π to ρ to σ should agree, where appropriate, with moving from π to σ directly. Failure of such compatibility indicates an obstruction.

This is why quantum gravity should be viewed as a gluing problem. Quantum theory and general relativity each supply powerful local quotient descriptions. But their transition maps are incomplete. Semiclassical gravity, quantum field theory in curved spacetime, effective quantum gravity, holography, and canonical approaches each provide partial transitions. None yet provides a universally accepted admissibility policy under which quantum and gravitational identity become fully compatible. The missing structure is not merely a new variable. It is a lawful transition system among policies.

2.5 Why identity must be indexed before physics can glue

Gluing requires local agreement. But agreement is meaningless unless one first states the identity relation under which agreement is judged. Two descriptions cannot be said to match unless the theory specifies what counts as the same observable, the same state, the same geometry, the same event, the same causal relation, or the same physical process. Therefore identity must be indexed before physics can glue.

The indexing set is the policy space Π . Each $\pi \in \Pi$ determines a quotient world $[X]\pi$. The family $\pi \mapsto [X]\pi$ is not a collection of arbitrary perspectives. It is a structured system of admissible reductions, refinements, transitions, and overlaps. A physical theory becomes admissibility-covariant when it can describe how its objects transform across Π without pretending that one policy is absolute in every regime.

This is analogous to covariance in geometry, but applied at a higher level. Ordinary covariance says that laws should not depend on arbitrary coordinate choices. Admissibility covariance says that physical identity should be transportable across lawful policies of description. A theory is admissibility-covariant when it can explain which quotient presentations are compatible, which are incompatible, and which invariants survive changes in policy.

Quantum theory and general relativity cannot be glued by assuming from the beginning that one identity policy dominates the other. If quantum identity is made absolute, then dynamical spacetime becomes difficult to accommodate. If diffeomorphic geometric identity is made absolute, then conventional quantum time evolution and measurement become difficult to formulate. If both are retained without indexing, contradiction or ambiguity results. The remedy is to treat each as valid over a region of Π and then search for transition maps, overlap conditions, and obstruction classes.

In this view, a classical spacetime is not the primitive arena of all reality. It is a stabilized quotient object, robust under many admissibility policies. A quantum state is not a ghostly object floating outside geometry. It is a contextual quotient object whose identity depends on algebraic, measurement-theoretic, and dynamical policies. A full theory of quantum gravity would not erase either structure. It would explain how both arise as quotient presentations of a deeper admissibility-covariant system.

The philosophical consequence is substantial but disciplined. Reality is not reduced to observer preference, language, or convention. Rather, reality is expressed through invariants that survive lawful admissible transformation. The more widely a quotient presentation glues across policies, the more classical and stable it appears. The more it fails to glue, the more contextual, quantum, or obstructed it appears. Physics becomes the study of lawful survival under admissible change.

The mathematical consequence is equally direct. Before searching for the final equation of quantum gravity, one must identify the admissibility structure in which such an equation would be meaningful. What are its presentations? What are its policies? What are its quotient objects? What are its invariants? What are its transition maps? What are its obstruction classes? What does it mean for two descriptions to be the same physical description when one is quantum and the other geometric?

Section 2 has established the working language for the rest of the paper. A presentation X is transformed by a policy π into a quotient $[X]\pi$. The invariant $\text{Inv}\pi(X)$ is what survives. Compatibility between policies is expressed through admissible transition. Gluing is possible only when quotient presentations agree on overlaps. Obstruction appears when they do not. Quantum gravity, in this framework, is the search for the admissibility-covariant structure in which quantum and gravitational quotient identities can be lawfully compared.

3. Quantum Theory as Contextual Quotient Physics

Quantum theory is usually introduced as a theory of states, observables, probabilities, and measurement. A system is represented by a state vector, density operator, wavefunction, path integral, algebraic state, or other formal presentation. Observables are represented by operators, effects, projectors, or algebraic elements. Measurements yield outcomes according to probability rules. Time evolution is unitary in closed systems and nonunitary in measurement-like or open-system descriptions. This is the familiar structure. Yet beneath it lies a deeper admissibility discipline. Quantum theory does not treat every mathematical distinction in a presentation as physically meaningful. It repeatedly quotients, contextualizes, or restricts identity.

A state vector is not identical with a physical state in the naive sense. Multiplication by a nonzero global phase changes the vector but not the physical ray. A mixed state may arise from many ensembles that are operationally indistinguishable. A subsystem may not possess an independent pure state when entangled with a larger system. An observable is not merely a number waiting to be read; it belongs to a context of compatible measurements. Noncommuting observables cannot generally be assigned simultaneous sharp values. A measurement outcome is not simply the revelation of a prewritten classical property; it is the admission of a presentation into a quotient determined by an experimental policy.

Admissibility Algebra therefore reads quantum theory as contextual quotient physics. A quantum presentation X becomes an experimentally or algebraically meaningful object only under a policy π that specifies the admissible observables, equivalences, preparations, transformations, coarse-grainings, and measurement contexts. The quotient $[X]\pi$ is the quantum object as available under π . Invariants are those features that survive the policy: transition probabilities, spectra, expectation values, correlation functions, algebraic relations, entanglement structure, conserved quantities, or contextual outcome distributions. Quantum ambiguity arises when no single policy-independent quotient can contain all such features simultaneously.

This interpretation does not replace quantum theory. It clarifies its identity layer. Quantum mechanics already teaches that the same system may have different legitimate descriptions depending on basis, observable algebra, subsystem decomposition, measurement arrangement, and environmental coupling. Admissibility Algebra makes this explicit by asking what policy licenses each description and how different quotient presentations may or may not glue.

3.1 States as presentations

A quantum state is often treated as the central object of the theory, but even the word state conceals multiple levels of structure. A ket, wavefunction, density matrix, path-integral amplitude, algebraic state, tensor network, or reduced density operator may each present quantum information in a different way. These presentations are useful, but none should be confused with raw physical identity without specifying the admissibility policy under which it is being interpreted.

A vector ψ in a Hilbert space is a presentation. The physical pure state is not ψ itself but its ray, because ψ and $e^{i\theta}\psi$ give the same physical predictions for ordinary quantum observables. Thus the admissibility policy π_{phase} identifies vectors related by global phase. The quotient $[\psi]\pi_{\text{phase}}$ is the ray. The invariant is not the vector as written, but the projective structure that determines transition probabilities and expectation values. This is already an elementary instance of quotient physics.

A density operator ρ is also a presentation, but one with a different admissibility role. It may describe classical ignorance about a preparation, entanglement with an inaccessible environment, or an operational equivalence class of preparation procedures. Distinct ensembles can produce the same density operator. Under the policy of operational indistinguishability, such ensembles are identified. The quotient is not the hidden list of possible preparations, but the state as tested by the available observables.

A reduced density operator deepens this point. If a system A is entangled with system B, the state of A alone is obtained by tracing over B. The operation discards distinctions in the composite presentation that are inaccessible to the local policy for A. The result is not a complete presentation of the universal state; it is a quotient presentation relative to subsystem access. The local object $[X]\pi_A$ preserves the statistics available to A while forgetting correlations that require access to B.

Thus a quantum state is not simply a thing. It is a presentation whose physical identity depends on the policy governing phase, access, preparation, subsystem decomposition, and measurement. Different policies produce different quotient states. These quotient states may be compatible, partially compatible, or incompatible. The mistake is to demand one unindexed object that carries every description without loss, context, or obstruction.

In quantum gravity this becomes critical. If a state is defined relative to a fixed time, fixed Hilbert-space factorization, or fixed background geometry, then the presentation already assumes part of what gravity makes dynamical. A state of geometry cannot simply inherit the identity policy of ordinary quantum mechanics without modification. One must first ask: under what admissibility policy is this state being presented, and what distinctions are preserved by that policy?

3.2 Observables as policy-relative invariants

An observable is not merely a mathematical operator. It is an invariant selected by an admissibility policy. To call something observable is to say that it survives the relevant quotient and can be lawfully compared across admissible presentations. In ordinary quantum mechanics, observables are represented by self-adjoint operators or more general measurement structures. But the operator alone does not determine the full physical meaning. One must also know the measurement context, domain, preparation, compatibility relations, and admissible transformations.

If a policy π selects a commuting family of observables, then the corresponding quotient $[X]\pi$ allows a classical-like assignment of outcomes within that context. If another policy ρ selects a different noncommuting family, then $[X]\rho$ may be equally valid but not jointly globalizable with $[X]\pi$. The quantum object is therefore not exhausted by either quotient alone. It is

expressed through a family of contextual quotients whose overlap structure encodes quantum behavior.

The invariant $\text{Inv}\pi(X)$ may be a spectrum, an expectation value, a probability distribution, a correlation function, or an algebraic relation. These invariants are not arbitrary. They are constrained by the Born rule, operator algebra, symmetry, dynamics, and experimental reproducibility. Yet they remain policy-relative because different policies select different questions that can be asked without contradiction.

This is clearest for incompatible observables. Position and momentum, different spin components, or noncommuting field observables cannot be assigned simultaneous sharp classical values in the usual formalism. From the admissibility viewpoint, the obstruction arises because the quotient appropriate for one observable context does not refine into a single global quotient preserving all contexts as if they were jointly classical. Each context is locally lawful. Their forced globalization is not.

The observable therefore becomes a test of admissibility. It asks whether a distinction in the presentation survives the policy. If it does, the distinction is physical under π . If it does not, the distinction is representational, inaccessible, or intentionally quotient-forgotten under π . The same underlying presentation may support many observable structures, but not all can be simultaneously stabilized into one classical identity scheme.

This point is essential for quantum gravity because gravitational observables are themselves constrained by diffeomorphism invariance. A coordinate-local field value may not be an observable in the gravitational sense unless it is defined relationally or invariantly. Quantum theory asks for operators and spectra; general relativity asks for diffeomorphism-invariant content. The quantum-GR gluing problem therefore includes the problem of identifying observables that survive both policies. The relevant invariant is not merely quantum and not merely geometric. It must be admissible under both.

3.3 Measurement as quotient selection

Measurement is the most visible place where quantum theory displays its contextual structure. In the traditional language, a measurement maps a quantum state into an outcome or outcome distribution. In some interpretations, it collapses the state. In others, it entangles system and apparatus, branches worlds, updates information, selects histories, or records correlations. Admissibility Algebra abstracts from these interpretive choices and describes measurement as quotient selection.

Before measurement, a quantum presentation may contain superposed amplitudes relative to a chosen basis or observable structure. The measurement policy π specifies an apparatus, a decomposition of possible outcomes, a coupling, a resolution, a pointer basis, an environment,

and a criterion for what counts as a record. The quotient $[X]\pi$ is the object as admitted into that outcome structure. Distinctions not visible to the measurement policy are discarded; distinctions amplified by the policy become outcome-relevant.

This does not mean that measurement is merely subjective. The policy π is embodied by a physical arrangement. It includes actual interactions, boundary conditions, apparatus stability, environmental decoherence, calibration, and operational access. A measurement policy is admissible only if it produces reproducible and lawful outcome statistics. The policy is contextual, but not arbitrary.

Collapse can therefore be read as admission into a quotient. The pre-measurement presentation X is not simply destroyed; rather, the theory replaces X , for the purpose of the measurement record, by $[X]\pi$. The quotient preserves the outcome-relevant invariant and discards distinctions that the measurement record cannot retain. In open-system language, this may correspond to decoherence, tracing out environmental degrees of freedom, or conditioning on a record. In operational language, it corresponds to updating the admissible description after an outcome.

Different measurement policies produce different quotient selections. A spin measurement along one axis selects a different quotient than a spin measurement along another. An interference experiment preserves phase distinctions that a which-path measurement destroys. A high-resolution measurement preserves distinctions that a coarse-grained measurement ignores. A weak measurement preserves a different structure than a projective measurement. Each policy defines what counts as a result.

The quantum measurement problem persists because one tries to describe all policies from within a single unproblematic identity frame. But if the identity frame itself is policy-indexed, the question changes. Instead of asking how one absolute quantum state becomes one absolute classical outcome, one asks how a presentation X is transformed into a family of quotient records $[X]\pi$, and under what conditions those records stabilize across observers, apparatuses, and environments.

For quantum gravity this reformulation is crucial. A measurement requires spacetime localization, causal order, and record formation. But in a gravitationally dynamical setting, localization and causal order may themselves be quotient-relative. The measurement policy cannot simply be imported from laboratory quantum mechanics. It must be enlarged to include geometric admissibility. Measurement in quantum gravity is therefore not only a quantum quotient selection; it is a quantum-geometric quotient selection.

3.4 Phase, entanglement, and unitary admissibility

Three features of quantum theory show especially clearly that admissibility is already built into the formalism: phase, entanglement, and unitarity.

Global phase is the simplest case. The transformation $\psi \mapsto e^{i\theta}\psi$ changes the presentation but not ordinary physical predictions. It is therefore admissible under the phase policy. Relative phase, by contrast, is often physically meaningful because it affects interference. The admissibility rule is not that phase never matters. It is that some phase distinctions survive the relevant policy while others do not. This is precisely the kind of disciplined distinction Admissibility Algebra is designed to express.

Entanglement shows that the identity of a part may depend on the whole. In a separable classical picture, a composite object is built from independently specified parts. In quantum theory, a composite state may be well defined while its subsystems possess only reduced or mixed descriptions. The subsystem identity is obtained by a quotient operation that discards inaccessible correlations. The full entangled presentation contains structure not visible in the local quotient. Therefore $[X]\pi_{\text{local}}$ and $[X]\pi_{\text{global}}$ are related but not identical.

This is not merely a technical feature. It changes the meaning of physical objecthood. A particle, subsystem, field mode, or laboratory region may not carry complete independent identity under all policies. Its identity may be relational, entangled, or context-dependent. The more entangled the presentation, the less adequate it becomes to treat the subsystem quotient as an autonomous absolute object.

Unitarity provides another admissibility condition. Closed quantum evolution preserves inner products, probabilities, and reversible structure. Under a unitary policy, admissible transformations are those that preserve the quantum information encoded in the Hilbert-space structure. This policy is powerful, but it is not the only policy operating in physics. Open-system dynamics, measurement, coarse-graining, thermodynamic irreversibility, and gravitational horizons all complicate the meaning of preserved information. The question becomes: under which policy is information preserved, and under which quotient is it inaccessible, scrambled, coarse-grained, or lost?

The tension between unitarity and gravitational phenomena is one of the deep signals of the quantum-GR obstruction. Black holes, horizons, singularities, and cosmological boundaries challenge simple assumptions about global quantum evolution. If unitarity is treated as an absolute identity policy, gravitational causal structure becomes difficult. If gravitational quotienting is treated as absolute, quantum information preservation becomes difficult. Admissibility Algebra does not solve this by fiat, but it gives a more precise question: what

transition rule relates unitary quantum admissibility to horizon or diffeomorphic gravitational admissibility?

Phase, entanglement, and unitarity therefore reveal quantum theory as a layered quotient system. It preserves some distinctions, discards others, and makes still others contextual. A future quantum gravity theory must preserve this subtlety. It cannot simply replace quantum identity with geometry, nor geometry with quantum statehood. It must identify a larger admissibility structure in which both forms of identity appear as lawful quotients.

3.5 Quantum ambiguity as failure of policy-independent identity

Quantum ambiguity is often described in terms of superposition, uncertainty, contextuality, or indeterminacy. Admissibility Algebra describes it as failure of policy-independent identity. The quantum object does not supply one global classical presentation whose properties can be read off under every measurement policy. Instead, it supplies a structured family of quotient presentations, each valid under a specific context, with obstructions to gluing them into one classical whole.

This does not mean that quantum theory is incoherent. On the contrary, its coherence lies in the precise laws governing incompatible contexts. The Born rule, operator algebra, commutation relations, tensor products, symmetry principles, and dynamical equations strictly constrain the family of possible quotients. Quantum ambiguity is not disorder. It is lawful non-globality.

A classical object is one whose quotient presentations agree across many policies. Its position, shape, mass, trajectory, and persistence remain stable under a broad family of observations and coarse-grainings. It can be reidentified by different observers under different conditions. Its local quotient presentations glue into a robust object. This is why classical reality feels policy-independent: many policies agree strongly enough that the indexing becomes invisible.

A quantum object does not generally have this property. Different contexts expose different invariants. Attempting to combine them into a single classical assignment produces contradiction or impossibility. The failure is not that the quantum object lacks reality, but that its reality is not carried by one policy-independent quotient. Its identity is distributed across contextual admissible presentations.

This interpretation aligns naturally with the gluing language needed for quantum gravity. In a sheaf-like formulation, each context provides local data. Classicality corresponds to successful globalization. Quantum contextuality corresponds to obstruction. The quantum state is then not a hidden classical object waiting behind the contexts, but a rule-governed structure whose local quotient presentations cannot all be glued into a single classical section.

The same pattern may occur for spacetime itself. Classical spacetime may be a regime in which geometric quotient presentations glue across many policies: observers, coordinate systems, scales, matter probes, and causal measurements. At quantum-gravitational scales, such gluing may fail. There may be no single policy-independent geometry, just as there is no single policy-independent assignment of noncommuting quantum values. Spacetime ambiguity would then be the gravitational analogue of quantum contextuality.

This is the bridge to the rest of the paper. Quantum theory teaches that physical identity may be contextual, quotient-based, and non-global. General relativity teaches that geometric identity is diffeomorphic, relational, and background-independent. Quantum gravity requires these lessons to be combined. The next step is to examine general relativity as its own quotient theory, not as a classical stage upon which quantum theory merely acts.

4. General Relativity as Diffeomorphic Quotient Geometry

General relativity is often described as a geometric theory of gravitation. Matter and energy determine curvature; curvature determines the motion of matter and light. Spacetime is no longer a passive stage but a dynamical structure. The metric field encodes distances, durations, causal cones, geodesics, curvature, and gravitational interaction. This is the standard account. Yet beneath the geometric language lies a profound admissibility principle: general relativity does not treat every mathematical distinction in a spacetime presentation as physically meaningful. It quotients by diffeomorphism.

This means that general relativity is not simply a theory of metric tensors written on manifolds. It is a theory of equivalence classes of geometric presentations under transformations that preserve physical content. Coordinates do not name physical reality by themselves. A manifold point has no absolute physical individuality apart from the relational field structure in which it appears. Two metric-field configurations related by an allowed diffeomorphism may be distinct as presentations and identical as physical geometries. General relativity is therefore already an admissibility theory of spacetime identity.

This is one of the deepest lessons of modern physics. The gravitational field is not an ordinary field living on an independently fixed arena. It participates in defining the arena. The admissible transformations of the theory do not merely move objects through space; they transform the mathematical representation of spacetime itself. A coordinate change, active diffeomorphism, or equivalent geometric presentation may alter the descriptive form while preserving the physical object. Thus general relativity forces a distinction between presentation geometry and quotient geometry.

Admissibility Algebra captures this distinction directly. A spacetime presentation G , consisting of a manifold, metric, matter fields, and additional geometric structure, is not yet the physical gravitational object. The physical object is $[G]\pi_{\text{diff}}$, the quotient of G under the admissibility policy of diffeomorphic equivalence and covariance. The invariant content $\text{Inv}\pi_{\text{diff}}(G)$ is not the coordinate expression of the metric, but the relational, causal, curvature, and dynamical structure that survives admissible transformation. This section develops general relativity as diffeomorphic quotient geometry and prepares the later comparison with quantum quotient physics.

4.1 Spacetime fields as presentations

A spacetime model in general relativity is commonly represented by a differentiable manifold M equipped with a Lorentzian metric g and appropriate matter fields Φ . Symbolically one may write a presentation as:

$$G = (M, g, \Phi)$$

This presentation contains enormous mathematical information. It includes manifold points, coordinate charts, tensor components, metric coefficients, curvature tensors, field values, causal cones, geodesic structure, and boundary or asymptotic data when present. But not all of this information is physical in the same sense. Some of it is representational scaffolding. Some is invariant content. General relativity is the discipline that tells us which is which.

A coordinate chart is a presentation device. It assigns numerical labels to regions of M . The numerical labels are useful for calculation, but they are not themselves physical objects. If one writes the same geometry in another coordinate system, the tensor components may change while the physical spacetime remains the same. Thus the coordinate expression of the metric is not the physical geometry. It is one presentation of it.

More deeply, even the manifold points themselves cannot be treated as independently individuated physical atoms. In a background-independent theory, the identity of a point is not given before the fields. It is fixed only through relational structure. A point is physically meaningful through its role in the metric, matter configuration, causal order, curvature relations, and observables that can be defined without arbitrary labels. The manifold supplies a differentiable support for the presentation, but physical identity lives in the quotient structure.

This can be stated in admissibility language. A spacetime presentation G becomes a physical gravitational object only after applying a policy π_{GR} . This policy identifies presentations that differ only by admissible relabeling or diffeomorphic transformation. The quotient $[G]\pi_{\text{GR}}$ is the geometry as physics, not merely as notation.

The significance is considerable. General relativity does not describe gravitational reality as a set of fields placed on a fixed background with externally labeled points. It describes relational geometry whose identity is preserved across admissible changes of presentation. The gravitational field is therefore not reducible to its coordinate expression, nor to a pointwise list of tensor components. It is the quotient object that survives covariance and diffeomorphism invariance.

This is why general relativity is structurally close to Admissibility Algebra. It already teaches that sameness in physics is not equality of presentations. The metric components may differ. Coordinates may differ. Manifold labels may differ. Yet the physical geometry may be the same. What matters is the invariant relational content preserved by the admissibility policy.

4.2 Diffeomorphism invariance as admissible identity

Diffeomorphism invariance is often described as a symmetry principle. It says, roughly, that the physical content of the theory is invariant under smooth transformations of the manifold that preserve the differentiable structure. But in general relativity this principle is more than a convenient symmetry. It is an identity rule. It tells us when two geometric presentations represent the same physical spacetime.

Let $f: M \rightarrow M$ be an allowed diffeomorphism. Acting on the metric and fields produces a transformed presentation:

$$G' = f^*G$$

The original presentation G and the transformed presentation G' may differ as mathematical expressions. Their tensor components may appear different in a chosen coordinate description. Their point labels may be shuffled. But under the diffeomorphic admissibility policy, they are physically equivalent:

$$G \equiv_{\pi_diff} G'$$

The quotient $[G]_{\pi_diff}$ is the equivalence class of all such admissible presentations.

This is not a minor representational detail. It is what prevents coordinates from becoming false physical structure. If one mistakenly treats each diffeomorphic presentation as a distinct universe, one multiplies entities beyond physical necessity. The theory would then contain spurious distinctions generated by description rather than reality. Diffeomorphism invariance removes those distinctions by quotienting them away.

At the same time, the quotient must not remove real structure. Two spacetimes with different curvature invariants, causal structures, singularity behavior, horizon structure, or relational matter configurations may not be physically equivalent. The admissibility policy is exacting. It

ignores only those differences that fail to survive diffeomorphic comparison. It preserves the relational geometry.

Diffeomorphism invariance therefore functions as a disciplined admissibility policy. It answers the question: what may be changed without changing the gravitational object? Coordinates may be changed. Labels may be changed. Presentational placement may be changed. But invariant causal and geometric relations must remain.

This has direct consequences for quantum gravity. If quantum theory is to glue with general relativity, it must respect this identity discipline. A quantum description of geometry cannot treat manifold labels, coordinate-local values, or background structures as automatically physical unless it explains how they survive diffeomorphic quotienting. Conversely, a gravitational description that discards too much may erase quantum distinctions necessary for interference, entanglement, or information preservation. The gluing problem is therefore not simply to put hats on metric components. It is to reconcile quantum admissibility with diffeomorphic admissibility.

4.3 Geometry, covariance, and background independence

General covariance states that the laws of physics should take a form independent of arbitrary coordinate choice. In its mature gravitational form, this is linked to background independence: the geometry of spacetime is not fixed in advance but participates dynamically in the physical system. The metric is not a passive bookkeeping device. It is a field with its own equations, constraints, and interaction with matter.

Background independence is a radical admissibility principle. It denies ultimate physical status to structures that many other theories presuppose. In ordinary nonrelativistic quantum mechanics, time is commonly treated as an external parameter. In special relativistic quantum field theory, Minkowski spacetime supplies a fixed causal and metric background. In general relativity, such background structure becomes dynamical. The theory does not begin with an immutable geometry and then add matter. It solves for geometry and matter together.

The admissibility consequence is that identity cannot be assigned by reference to an external spacetime scaffold. One cannot simply say that an event is the same because it occurs at the same coordinate point in a fixed background. The coordinate point has no independent physical authority. The event must be specified relationally, through fields, causal relations, curvature, boundary data, or other invariant structures.

Covariance removes dependence on arbitrary presentation. Background independence removes dependence on a fixed arena. Together they make general relativity a quotient theory of dynamical relational geometry. The physical content of the theory lies not in a field over a predetermined stage, but in the admissible equivalence class of the stage-field structure itself.

This creates immediate pressure on quantum theory. Standard quantum formalism often assumes a background time parameter for evolution, a Hilbert-space structure defined over a given configuration space, and observables specified relative to a background causal or spatial decomposition. These assumptions are enormously successful in ordinary regimes, but they conflict with the admissibility discipline of full general relativity. If spacetime geometry is dynamical, then the background structures used to define quantum identity may themselves require quotienting.

The deeper issue is therefore not that geometry is classical and quantum theory is probabilistic. The issue is that quantum theory often begins with background structures that general relativity refuses to treat as absolute. Any quantum-GR gluing must therefore explain how quantum states, observables, and time evolution can be formulated without violating gravitational admissibility. It must also explain how classical geometric structure can emerge as a stable quotient where many policies agree.

Background independence can be restated in the language of Admissibility Algebra:

No presentation-dependent spacetime structure may be treated as physically primitive unless it survives the diffeomorphic admissibility policy.

This principle is not optional for general relativity. It is the theory's identity law. Therefore any proposed quantum gravity theory must either preserve it, derive it, or explicitly explain why and where it fails.

4.4 Observers, frames, and local admissibility

General relativity does not eliminate observers, frames, or local measurements. It disciplines them. An observer may choose a coordinate system, carry a clock, define a local tetrad, trace a worldline, or measure proper time and local fields. These are valid local presentations. But the theory does not confuse the observer's frame with absolute spacetime structure.

A frame is a policy. It determines what quantities are directly accessible to a given observer, apparatus, or local region. It selects a decomposition of spacetime into measurable quantities: time along a worldline, spatial directions in a local rest frame, redshift comparisons, tidal effects, accelerations, or local field values. These quantities are not arbitrary. They are constrained by the metric and matter fields. But they are still local and policy-relative.

General relativity therefore contains a hierarchy of admissibility. At the most general level, diffeomorphism invariance governs physical equivalence. At local levels, frames and observers define practical measurement policies. A freely falling observer, an accelerated observer, a distant asymptotic observer, and an observer near a horizon may assign different local

descriptions. The theory relates these descriptions through geometric laws rather than declaring one frame absolutely privileged.

This is crucial for the later quantum comparison. Quantum measurement is also policy-relative. It depends on apparatus, observable context, preparation, and accessible degrees of freedom. General relativity adds that the observer's spacetime frame is itself part of the relational geometry. A full quantum-GR theory must therefore combine measurement-context admissibility with frame and diffeomorphism admissibility.

Local admissibility becomes especially subtle near horizons, singularities, and cosmological boundaries. What one observer can access may differ from what another can access. Causal structure restricts information flow. Horizons divide regions of accessibility. Proper time may remain finite for one observer while coordinate descriptions become singular for another. These are not merely coordinate puzzles. They reveal that physical description depends on admissible access.

An observer policy π_{obs} may yield a quotient $[G]\pi_{\text{obs}}$ that preserves locally measurable quantities while discarding inaccessible global structure. Another policy π_{global} may preserve a different invariant family. A mature gravitational theory must explain how such quotient presentations relate. In classical general relativity this is handled through covariance, causal structure, and invariant geometric relations. In quantum gravity, the task is harder because quantum information, entanglement, measurement, and horizon structure must be included.

The lesson is that general relativity already treats observation as embedded in geometry. An observer is not outside spacetime looking in. The observer is a worldline, frame, apparatus, or local physical system within the dynamical structure. This means that admissibility policies in gravitational physics cannot be merely epistemic. They are physically instantiated in causal access, metric structure, and relational measurement.

4.5 The gravitational field as a quotient object

The gravitational field is not merely the metric tensor g written in a chosen coordinate system. It is the diffeomorphism-invariant quotient object represented by the metric and its relations to matter, causal structure, and curvature. In Admissibility Algebra notation, if $G = (M, g, \Phi)$ is a gravitational presentation, then the gravitational object is:

$[G]\pi_{\text{GR}}$

where π_{GR} includes diffeomorphic equivalence, covariance, relational observability, and background independence.

This quotient object preserves what general relativity regards as physically meaningful: causal cones, proper intervals, curvature relations, geodesic behavior, horizon structure, matter-

geometry coupling, and invariant relational facts. It discards coordinate labels, arbitrary chart choices, and presentation-dependent distinctions. The gravitational field is therefore not a thing sitting in spacetime. It is the admissible relational structure of spacetime itself.

This point becomes decisive when one tries to quantize gravity. If one begins with a metric presentation g and applies quantization mechanically, one risks quantizing nonphysical presentation degrees of freedom. Gauge constraints and diffeomorphism invariance then reappear as problems to be imposed, solved, or factored out. From the admissibility viewpoint, this indicates that the quotient identity of the gravitational field must be respected from the start. One should not quantize first and ask about physical identity later. The admissibility policy is part of the object.

The gravitational quotient also explains why time is difficult. In ordinary quantum theory, time is often an external parameter with respect to which states evolve. In general relativity, time is embedded in the metric and varies with gravitational structure. Proper time, coordinate time, causal ordering, and global time foliations are not equivalent notions. A gravitational field does not merely evolve in time; it helps define the temporal structure in which evolution is described. Therefore the quotient object $[G]\pi_{\text{GR}}$ includes time as relational geometry, not as an external universal parameter.

This creates one of the central quantum-GR obstructions. A quantum state typically needs an admissibility policy involving evolution, observables, and probability over time. A gravitational field supplies a policy in which time is dynamical and diffeomorphism-relative. These policies do not automatically glue. Their mismatch is not accidental. It follows from the identity structure of the two theories.

The gravitational field as quotient object also reframes spacetime emergence. If classical geometry is $[G]\pi_{\text{GR}}$, then a deeper theory need not contain classical spacetime as a primitive presentation. It may contain a more fundamental object X whose quotient under a coarse geometric policy yields $[G]\pi_{\text{GR}}$. Under another policy, the same X may yield a quantum algebraic object. Quantum gravity would then be the theory of X together with the admissibility policies that produce its quantum and geometric quotients.

Section 4 therefore establishes the gravitational half of the comparison. General relativity is not naive classical realism. It is already a sophisticated quotient theory. It rejects absolute coordinate identity, rejects fixed background structure, embeds observers within relational geometry, and treats the gravitational field as an equivalence class of admissible presentations. The quantum-GR problem is difficult precisely because quantum theory and general relativity are both quotient theories, but their quotient policies do not yet share a common gluing law.

5. The Quantum-GR Mismatch

The quantum-GR mismatch is often described as a clash between probabilistic microscopic physics and deterministic spacetime geometry. This description is useful but incomplete. The deeper mismatch concerns the admissibility conditions by which each theory defines states, observables, evolution, locality, and identity. Quantum theory and general relativity are not simply written in different mathematical languages. They decide differently what may be held fixed, what may be transformed, what may be ignored, what counts as a physical distinction, and what counts as an admissible object.

Quantum theory commonly begins with a state space, an algebra of observables, and an evolution rule. General relativity begins with a dynamical spacetime geometry in which the arena of events, distances, causal relations, and temporal intervals is itself part of the solution. Quantum theory asks how states evolve in time. General relativity asks how time is constituted by the metric and matter fields. Quantum theory often requires a background sufficient to define localization, tensor-factor decomposition, and measurement context. General relativity refuses to treat the background as a fixed physical primitive.

This does not mean either theory is wrong. It means that each theory is internally coherent under a different admissibility policy. Quantum theory preserves algebraic, probabilistic, phase, entanglement, and unitary structures. General relativity preserves diffeomorphic, causal, metric, and background-independent structures. The obstruction arises when one attempts to use both policies at once without a higher rule governing their overlap.

The mismatch therefore has the form of a failed gluing problem. On the quantum chart, identity is encoded by rays, operators, spectra, amplitudes, correlations, unitary transformations, and measurement policies. On the gravitational chart, identity is encoded by diffeomorphism classes, causal relations, curvature, proper intervals, and relational observables. The question is not simply how to write a quantum equation for the metric. The question is how to define a quotient object that is simultaneously admissible under quantum and gravitational identity rules.

5.1 External time versus dynamical time

The most famous quantum-GR mismatch concerns time. In ordinary quantum mechanics, time is usually an external parameter. A state evolves according to a rule that presupposes a temporal ordering. One writes a state at time t , evolves it to another time, computes transition probabilities, and predicts measurement statistics. Even when the formalism becomes relativistic, one typically assumes enough background causal structure to define evolution, locality, commutation relations, and field propagation.

In general relativity, time is not an external universal parameter. Time is part of the dynamical geometry. Proper time depends on the metric along a worldline. Coordinate time depends on a chosen chart or foliation. Gravitational fields affect clock rates. Spacetime may not admit a preferred global slicing. In strong gravitational regimes, horizons, singularities, and topology may further complicate any simple temporal ordering. Time is therefore not a fixed stage on which physics unfolds. It is part of the field structure.

This is not merely a technical inconvenience. It is an admissibility conflict. Quantum theory commonly uses a policy π_Q in which temporal evolution is defined relative to a background parameter or causal structure. General relativity uses a policy π_{GR} in which that temporal structure is itself subject to diffeomorphic equivalence and dynamical variation. What is fixed under π_Q is variable under π_{GR} . What functions as an external ordering condition in one theory becomes an internal relational feature in the other.

The problem becomes acute in canonical approaches to quantum gravity, where constraints replace ordinary time evolution and the notion of a universal external time disappears. It also appears in cosmology, where the universe as a whole has no external clock standing outside it. It appears near black holes, where different observers may assign different temporal descriptions to horizon-crossing processes. It appears in semiclassical gravity, where quantum matter evolves on a spacetime background that is itself only approximately classical.

In admissibility language, the issue is this: the quotient $[X]\pi_Q$ may require a time parameter to define state evolution, while the quotient $[X]\pi_{GR}$ treats time as part of the quotient geometry. These quotient objects do not automatically compare. A transition map between them must explain how quantum evolution is recovered when time is relational, approximate, emergent, or policy-dependent.

This suggests that the final theory cannot simply retain ordinary time and add quantum geometry to it. Nor can it simply dissolve quantum dynamics into timeless geometry. It must identify the admissibility conditions under which time behaves as an external parameter, when it behaves as a relational observable, when it becomes approximate, and when it fails to globalize. The quantum-GR glue requires a theory of temporal admissibility.

5.2 State evolution versus geometric constraint

Quantum theory is organized around states and their transformations. A state may evolve unitarily, collapse relative to a measurement description, decohere through interaction with an environment, or transform under a symmetry. The state is the carrier of probabilities, amplitudes, correlations, and dynamical information. Even when the formalism is generalized, the idea of a state remains central.

General relativity is organized around geometric constraint. The metric and matter fields must satisfy relations imposed by covariance, curvature, stress-energy, and diffeomorphism invariance. The gravitational degrees of freedom are not freely evolving in an external time. They are constrained by the structure of spacetime itself. Coordinate descriptions may suggest evolution, but the full theory contains redundancy associated with the choice of coordinates and slicing.

This creates a second admissibility conflict. In the quantum policy, physical description is often state-centered. In the gravitational policy, physical description is constraint-centered and quotient-centered. A quantum state may seem to specify what exists at a time. A gravitational solution specifies an entire relational spacetime structure subject to constraints. The two pictures are not trivially equivalent.

If one attempts to quantize geometry as if it were an ordinary field on a fixed background, diffeomorphism constraints immediately demand attention. Some degrees of freedom in the metric presentation are not physical; they are gauge or coordinate artifacts. Quantizing them without first respecting the quotient structure risks creating spurious states. Conversely, if one imposes gravitational constraints too rigidly before recovering quantum dynamics, one may obscure the probabilistic and operational content that ordinary quantum theory requires.

The mismatch is therefore not only between equations but between roles. In quantum theory, the state often functions as the primary presentation. In general relativity, the full geometric solution modulo diffeomorphism functions as the physical object. The quantum formalism asks for a Hilbert space of possibilities. The gravitational formalism asks for equivalence classes of geometries satisfying constraints. A quantum-gravitational state must somehow be both: a carrier of quantum amplitudes and a diffeomorphism-admissible relational object.

Admissibility Algebra reframes the problem. Let X be a deeper presentation. Under a quantum policy π_Q , X yields a state-like quotient $[X]_{\pi_Q}$. Under a gravitational policy π_{GR} , X yields a geometry-like quotient $[X]_{\pi_{GR}}$. The gluing question is whether there exists a transition structure relating these quotients without making either identity policy absolute. State evolution and geometric constraint may then be different quotient projections of a deeper admissibility-covariant object.

This does not solve the dynamics by itself. It clarifies what the dynamics must accomplish. It must preserve the quantum invariants required for amplitudes, probabilities, and correlations while also preserving the gravitational invariants required for covariance, constraint, and relational geometry. A theory that preserves only one side is not yet the glue.

5.3 Operator identity versus diffeomorphic identity

Quantum theory and general relativity also disagree about observables. In quantum theory, observables are represented by operators, effects, projectors, algebraic elements, or related structures. Their identity is controlled by algebraic relations, spectra, commutation properties, domains, symmetries, and measurement contexts. Two observables may be equivalent if they produce the same operational statistics under the admissible preparations and measurements. The algebraic structure matters.

In general relativity, an observable must survive diffeomorphism invariance. A coordinate-local quantity is not automatically physical, because a coordinate label has no intrinsic physical authority. A gravitational observable must be relational, invariant, boundary-defined, asymptotic, or otherwise constructed so that it does not depend on arbitrary presentation. The geometric structure matters.

The mismatch appears when one asks what the quantum observables of gravity are. If one writes an operator corresponding to a metric component at a coordinate point, the gravitational policy asks whether that coordinate point is physically meaningful. If it is not, the operator may represent a presentation artifact rather than an admissible observable. But if one restricts only to fully diffeomorphism-invariant observables, one may lose ordinary local field-theoretic intuition. The local operator and the diffeomorphic observable belong to different admissibility regimes.

This conflict is especially sharp because quantum theory depends heavily on observable algebras. Commutation relations encode causal structure, compatibility, and measurable distinctions. In curved spacetime or quantum gravity, causal structure may itself be dynamical or emergent. Then the algebra of observables cannot simply be fixed independently of geometry. Yet geometry itself is to be described quantum mechanically. The operator algebra and the causal-metric structure appear to define each other.

Admissibility Algebra describes this as a failure of direct identity transfer. An operator O under policy π_Q is not automatically an observable under π_{GR} . A geometric invariant R under policy π_{GR} is not automatically an operator in the quantum sense. A gluing map must preserve what both policies require: algebraic measurability and diffeomorphic admissibility. The appropriate quantum-gravitational observable may therefore be neither a naive local operator nor a purely classical invariant, but a quotient construction stable under both policies.

The problem is not unique to gravity, but gravity intensifies it. Gauge theories already require that observables respect gauge invariance. General relativity makes the gauge structure spacetime itself. The admissibility policy is therefore more radical. The identity of where an observable is located may itself be relational. The question “what is the value of this field

here?” must be replaced by a relational question such as “what is the value of this field where this other physical condition is satisfied?” or “what invariant relation is preserved across admissible presentations?”

The operator-diffeomorphism mismatch thus reveals a deeper requirement: quantum gravity needs an admissibility theory of observables. Such a theory must state which operators are physical, which geometric quantities are measurable, which local notions are approximate, which boundary or relational constructions are fundamental, and how observable identity changes across policies.

5.4 Local measurement versus global covariance

Quantum measurement is typically local, operational, and apparatus-relative. An experiment is performed in a region. A detector couples to a system. A record is formed. An outcome is compared with a prediction. This structure presupposes enough localization and causal order to distinguish system, apparatus, environment, preparation, interaction, and record.

General covariance, by contrast, demands that physical laws not depend on arbitrary coordinate descriptions. In gravity, locality itself must be treated carefully because coordinate-local statements may not be physical. One may speak of local measurements by observers, but the theoretical object must remain invariant under diffeomorphic transformations. The local measurement must be embedded in a globally covariant structure.

The mismatch is not that local measurement is impossible in general relativity. Local measurements are essential. The mismatch is that quantum theory often treats the measurement context as sitting inside a fixed spacetime stage, whereas general relativity requires the stage to be dynamical and covariant. The laboratory frame, detector trajectory, timing, spatial region, and causal access are all gravitationally meaningful. They cannot be treated as external, absolute scaffolding in a full theory.

A measurement policy π_M may select an apparatus, pointer basis, outcome resolution, and environmental coupling. A gravitational policy π_{GR} may identify spacetime presentations related by diffeomorphism and require relational observability. A quantum-gravitational measurement policy must combine both: it must define outcomes in a way that is operationally meaningful and diffeomorphism-admissible.

This is particularly important in cosmology and black hole physics. In cosmology, there may be no external observer outside the universe. Measurement-like records must be internal correlations within the universe itself. In black hole physics, observers separated by horizons may have different access to information and different admissible descriptions. Global covariance and local accessibility may not align in a naive way. The same physical process may have distinct quotient presentations for different observer policies.

Local measurement and global covariance therefore form another gluing problem. The local quotient $[X]\pi_{\text{local}}$ preserves what an observer or apparatus can record. The covariant quotient $[X]\pi_{\text{cov}}$ preserves what survives diffeomorphic transformation. These are both legitimate, but they need transition rules. A local record must be expressible within a covariant theory, and the covariant theory must explain how local records arise.

Classical general relativity manages this through local frames, tensorial laws, and relational quantities. Ordinary quantum mechanics manages measurement through apparatus-dependent outcome statistics. Quantum gravity must manage both at once. It must explain how a measurement event is localized when localization is relational, how a record forms when geometry is dynamical, and how outcome probabilities are defined when the causal structure needed to define the experiment may itself be quantum or emergent.

The admissibility-covariant formulation avoids choosing between local measurement and global covariance. It treats each as a policy and asks how they glue. A valid quantum-gravitational theory must provide a sheaf of descriptions in which local records and global covariance are not rivals but compatible quotient presentations where compatibility is possible, and explicitly obstructed where it is not.

5.5 Why direct quantization does not remove the obstruction

Direct quantization attempts to apply quantum rules to gravitational variables. This is natural. If electromagnetism, matter fields, and other interactions can be quantized, perhaps the metric or connection can be quantized as well. Such approaches have produced important insights and mathematical structures. But direct quantization by itself does not remove the admissibility obstruction.

The reason is simple: quantization changes the formal status of variables, but it does not automatically solve the identity problem. If the variables being quantized include presentation-dependent degrees of freedom, then the quantum theory inherits redundant structure. Constraints, gauge fixing, ghosts, reduced phase spaces, relational observables, or other devices must then be introduced to recover the physical quotient. These methods may be necessary, but they show that admissibility cannot be postponed indefinitely.

Direct quantization also risks importing ordinary quantum identity too early. Standard quantization procedures often assume a background time, a phase space, a Hilbert space, canonical variables, or a fixed causal structure. But gravity questions precisely those structures. If the quantization procedure assumes the identity policy of nongravitational quantum theory, it may fail to respect background independence. If it fully imposes gravitational admissibility first, it may struggle to recover the operational quantum structures needed for measurement, probability, and evolution.

This does not mean quantization is wrong. It means quantization is not the whole problem. Quantization is a transformation of formal description. Gluing is a transformation of identity policy. Quantum gravity requires both. A theory may be mathematically quantum and still fail to be admissibly gravitational. Conversely, a theory may be geometrically covariant and still fail to preserve the quantum structures required for interference, entanglement, and probability.

The same critique applies to any one-sided program. Geometrizing quantum theory may fail if it erases noncommuting contextual structure. Quantizing geometry may fail if it preserves too much background-dependent presentation. Purely information-theoretic reconstructions may fail if they cannot recover diffeomorphic geometry. Purely geometric reconstructions may fail if they cannot recover quantum measurement and algebraic observables. The issue is not which language is fashionable, but which admissibility policy is being made absolute.

Admissibility Algebra proposes a different criterion. A candidate quantum gravity theory should be judged by whether it supplies lawful quotient presentations and transition maps among them. Can it recover quantum theory under policies where measurement, operator algebra, amplitudes, and entanglement are admissible? Can it recover general relativity under policies where diffeomorphism invariance, causal structure, and geometric covariance are admissible? Can it explain when these quotients glue and when they are obstructed? Can it state what identity relation replaces naive equality at the overlap?

This criterion does not eliminate the need for equations. It tells us what equations must be equations of. They must not merely describe a metric operator, a spin network, a path integral, a causal set, a boundary theory, or a wavefunction of the universe. They must describe an admissibility-covariant object whose quotient presentations reproduce the valid identity structures of both quantum theory and general relativity.

Section 5 has therefore sharpened the problem. External time conflicts with dynamical time. State evolution conflicts with geometric constraint. Operator identity conflicts with diffeomorphic identity. Local measurement conflicts with global covariance. Direct quantization does not automatically remove these conflicts because they are not only formal conflicts; they are quotient conflicts. The next step is to formulate this mismatch mathematically as a sheaf-theoretic gluing problem over policy space.

6. The Sheaf-Theoretic Formulation

The quantum-GR mismatch can be described conceptually as a conflict of admissibility policies, but a mature formulation requires mathematical structure. The natural structure is sheaf-theoretic. A sheaf begins from a simple idea: one may know an object locally without knowing whether those local descriptions assemble into a single global object. Local data may agree on

overlaps and therefore glue. Or they may fail to agree, revealing an obstruction. This is precisely the form of the quantum-GR problem.

Quantum theory provides local admissible descriptions: states, observables, measurement contexts, algebras, amplitudes, histories, and correlations. General relativity provides local admissible descriptions: coordinate charts, frames, causal neighborhoods, metric fields, curvature relations, diffeomorphism classes, and relational observables. Each theory knows how to regulate identity within its own domain. The problem is whether these descriptions can be glued into a deeper structure that preserves both quantum and gravitational invariants.

The sheaf-theoretic formulation treats admissibility policies as the base over which physical descriptions vary. Instead of assuming one universal object-space, one defines a policy space Π . Each region $U \subset \Pi$ corresponds to a family of admissibility conditions: quantum measurement contexts, gauge choices, diffeomorphic regimes, scale limits, observer frames, causal access restrictions, coarse-grainings, and boundary conditions. Over each such region, there is a category, algebra, or structured space of admissible physical presentations. The physical theory becomes a rule assigning local quotient descriptions to policy regions and specifying when they agree.

In this language, quantum gravity is not merely the search for one global equation. It is the search for a gluing law over Π . A theory succeeds when its quantum and gravitational quotient presentations form compatible local sections of a higher admissibility object. It fails, or remains incomplete, when the local sections cannot be glued without contradiction, loss of invariant content, or unlicensed identification. The obstruction is not an embarrassment. It is information. It tells us where the identity policies of physics refuse to globalize.

6.1 Policy space Π

The policy space Π is the space of admissibility conditions under which physical descriptions are formed. It is not ordinary spacetime. It is not Hilbert space. It is not configuration space. It is a higher indexing domain for the rules of physical identity. A point $\pi \in \Pi$ represents a policy: a specification of which distinctions are preserved, which transformations are admissible, which observables are meaningful, what scale is being used, what background or relational structure is allowed, and what counts as the same physical object.

Examples of policies include a choice of quantum measurement context, a commuting observable algebra, a subsystem decomposition, a gauge admissibility rule, a diffeomorphism-invariant geometric regime, a semiclassical approximation, a causal access condition, a renormalization scale, a coarse-graining map, a boundary reconstruction rule, or an observer-frame policy. These are usually treated as heterogeneous elements of different formalisms.

Admissibility Algebra places them into one structural role: each is a rule for quotienting presentations into physical objects.

Π need not be a smooth manifold. It may be a category, site, poset, higher category, simplicial object, type-theoretic universe, or stratified space. Its structure depends on the theory under construction. Some policies refine others. Some overlap. Some are incompatible. Some can be connected by transition maps. Some belong to different regimes separated by obstruction. The topology or coverage on Π records which policy families are allowed to count as local neighborhoods for gluing.

A simple ordering may be defined by refinement. A policy ρ refines π if ρ preserves all distinctions preserved by π and possibly more. Then π is coarser and ρ is finer. A classical coarse-graining policy may forget many microscopic distinctions; a quantum fine-grained policy may preserve phase and entanglement; a diffeomorphic policy may forget coordinate labels while preserving geometric relations. Refinement is not always linear. Two policies may preserve different incompatible distinctions, so neither refines the other.

This nonlinearity is essential. Quantum measurement contexts are often mutually incompatible. Gravitational coordinate charts may overlap but require nontrivial transformations. Boundary and bulk descriptions may preserve different structures. Effective theories at different scales may be related by renormalization but not by simple inclusion. Thus Π is not a ladder from ignorance to knowledge. It is a structured field of admissibility regimes.

The purpose of Π is to prevent premature absolutization. Rather than asking which policy is the true one, the formulation asks how policies relate. Which ones overlap? Which ones admit restriction? Which ones admit refinement? Which ones generate equivalent quotient objects? Which ones obstruct global identity? A physical theory over Π becomes a theory not only of objects, but of lawful movement among identity rules.

6.2 Local charts of admissible physical description

Given a region $U \subset \Pi$, one assigns a structured collection $\mathcal{F}(U)$ of physical descriptions admissible over U . This assignment may be a set, category, algebra, groupoid, ∞ -groupoid, type, stack, or other mathematical object, depending on the level of structure required. The key point is that $\mathcal{F}(U)$ contains descriptions valid under all policies in U , together with the equivalence and transformation rules those policies impose.

For a purely quantum region U_Q , $\mathcal{F}(U_Q)$ may contain Hilbert-space states, operator algebras, density matrices, measurement contexts, transition amplitudes, or algebraic states. Its admissibility rules identify global phase, preserve probabilities, encode commutation relations, and respect unitary or open-system dynamics. The quotient descriptions are quantum-contextual.

For a purely gravitational region U_{GR} , $\mathcal{F}(U_{GR})$ may contain metric-field configurations, diffeomorphism classes, causal structures, curvature data, relational observables, or geometric solutions. Its admissibility rules identify coordinate artifacts, preserve tensorial and causal relations, and respect covariance and background independence. The quotient descriptions are geometric-relational.

For an overlap region U_{QG} , if it exists, $\mathcal{F}(U_{QG})$ must contain descriptions that are simultaneously meaningful under quantum and gravitational policies. This is where the central difficulty lies. Such descriptions cannot depend on fixed background structures in a way forbidden by general relativity, nor can they discard quantum structures required for interference, entanglement, and probability. They must be quotient objects of a deeper kind.

The word chart is deliberate. A chart gives a local description of a more general object. No single chart is expected to exhaust the object globally. Quantum theory may be a chart. General relativity may be a chart. Semiclassical gravity may be a transition chart. Holographic duality may be a nonlocal chart relating bulk and boundary policies. Effective field theory may be a scale-dependent chart. The full object is not identical with any one chart; it is what is coherently presented by all admissible charts and their transition data.

Local charts also permit failure. There may be regions of Π where no current theory supplies valid descriptions. There may be overlaps where existing descriptions disagree. There may be transition maps that work perturbatively but fail nonperturbatively. There may be singular or boundary regimes where the admissibility policy itself degenerates. A sheaf-theoretic formulation does not hide these failures. It locates them.

Thus the assignment $U \mapsto \mathcal{F}(U)$ is a disciplined way to say: for this family of policies, here are the physical objects, invariants, transformations, and quotient identities available. Quantum gravity becomes the problem of extending this assignment so that quantum and gravitational charts belong to a coherent global structure, or of proving that certain obstructions are intrinsic.

6.3 Restriction maps, transition maps, and overlap conditions

A sheaf-like theory must specify how descriptions change when policies are restricted, refined, or compared. If $V \subset U$ is a smaller region of policy space, there should be a restriction map:

$$\text{res}_V^U: \mathcal{F}(U) \rightarrow \mathcal{F}(V)$$

This map sends a description valid under the broader policy family U to the description valid under the narrower family V . Restriction may forget structure, refine structure, localize structure, or reinterpret invariants. The precise direction depends on how Π is organized, but the conceptual role is stable: it explains how a description behaves when the admissibility conditions change.

For example, a semiclassical description may restrict to an approximately classical gravitational description when quantum fluctuations are ignored. A quantum field description may restrict to a local algebra associated with a smaller causal region. A geometric solution may restrict to a coordinate neighborhood or observer frame. A global thermodynamic description may restrict to a coarse macroscopic policy. These operations are familiar. The sheaf formulation collects them into one language.

Transition maps appear when two local descriptions overlap. Suppose U and V are regions of Π with overlap $U \cap V$. A description $s_U \in \mathcal{F}(U)$ and a description $s_V \in \mathcal{F}(V)$ are compatible when their restrictions to $U \cap V$ agree under the admissibility rules of the overlap:

$$\text{res}_U^{\{U \cap V\}}(s_U) = \text{res}_V^{\{U \cap V\}}(s_V)$$

The equality here should not be naive equality of presentations. It means admissible equality in $\mathcal{F}(U \cap V)$. Two descriptions agree on the overlap if they preserve the same quotient object there. In quantum-GR language, a quantum description and a gravitational description agree only if their overlap invariants coincide under a policy that can legitimately compare them.

This condition is subtle. A quantum description may contain phase information not visible in a classical geometric description. A gravitational description may contain causal structure not fixed in the quantum presentation. Agreement on the overlap cannot require both descriptions to be identical in all features. It requires that the features admissible in the overlap match. The overlap policy decides which invariants are comparable.

Transition maps must also satisfy coherence. If descriptions are passed from U to V to W , the result should agree with passing directly from U to W where such comparison is admissible. In category-theoretic language, this is functorial or natural behavior. In sheaf language, it is compatibility of restrictions. In physics, it is the requirement that changing descriptions does not generate inconsistent physical content.

Failure of overlap agreement marks obstruction. If the quantum and gravitational restrictions disagree on an invariant both are required to preserve, gluing fails. If one description requires a structure the other forbids, gluing fails. If a transition exists only after discarding essential quantum or gravitational content, the transition is not admissible. A false glue is worse than no glue, because it creates apparent unification by erasing the very invariants that made the theories successful.

6.4 Gluing compatible quotient presentations

The gluing axiom states that if one has local descriptions that agree on all overlaps, then there exists a global description whose restrictions recover the local ones. In the present setting, suppose a policy region U is covered by smaller regions $\{U_i\}$. For each U_i , let $s_i \in \mathcal{F}(U_i)$ be a local

quotient presentation. If for every overlap $U_i \cap U_j$ the restrictions of s_i and s_j agree admissibly, then there should exist a global section $s \in \mathcal{F}(U)$ such that:

$$\text{res}_{U_i}^U(s) = s_i$$

for every i .

This is the mathematical form of lawful physical unification. The global object s is not imposed by rhetoric. It exists only if the local quotient presentations agree in the required way. If they do, then the theory has successfully glued the descriptions. If they do not, the failure identifies an obstruction.

In quantum gravity, the local sections might include a quantum-field description in a weakly curved region, a classical geometric description in a low-energy regime, a thermodynamic horizon description, a boundary algebra description, or an effective gravitational description at a chosen scale. The global question is whether these can be restrictions of one admissibility-covariant object. The answer may be yes in some regimes, no in others, and partially yes under additional refinement.

Classical spacetime can be interpreted as a successful gluing of many local geometric quotient presentations. Different coordinate charts, local frames, and observer descriptions agree on overlaps through tensorial transition laws and diffeomorphic equivalence. This is why a manifold with metric structure can function as a global object. Its local presentations glue.

Quantum contextuality can be interpreted as a failure of classical gluing. Local measurement contexts provide valid outcome structures, but they cannot always be assembled into one global assignment of preexisting values. The local data are real and lawful; the global classical section does not exist. Quantum theory is therefore already a sheaf-theoretic warning against naive globalization.

Quantum gravity combines both patterns. It seeks a global structure deeper than classical spacetime, but it must respect the possibility that some classical globalizations fail at quantum-gravitational scales. The correct global object may not be a classical manifold, a single Hilbert space, or a fixed causal order. It may be a stack, higher sheaf, or admissibility-indexed family of quotient objects whose local presentations include both quantum and geometric regimes.

Gluing compatible quotient presentations also clarifies emergence. Emergence is not magic. It occurs when local or fine-grained descriptions stabilize into a coarser quotient that glues across many policies. A classical object emerges when different observations, frames, scales, and environmental couplings produce compatible quotient presentations. Classical spacetime emerges when geometric descriptions become robust across enough admissibility regions. The more widely a quotient glues, the more objective it appears.

6.5 Obstruction classes and failed globalization

Not all local data glue. When they fail, the failure can often be measured by an obstruction. In geometry, obstruction classes may prevent local trivializations from assembling globally. In topology, local patches may be compatible only up to nontrivial transition data. In quantum contextuality, local value assignments may fail to extend to a global assignment. In gauge theory, nontrivial bundles reveal that no single global gauge potential captures the whole structure without patching. These failures are not defects of mathematics. They are discoveries of structure.

The quantum-GR obstruction should be treated in the same way. If quantum and gravitational descriptions fail to globalize, the failure may encode the deep structure of quantum gravity. Instead of asking only how to eliminate the obstruction, one should ask what it measures. Does it measure failure of global time? Failure of fixed causal order? Failure of local operator identity? Failure of diffeomorphic observability? Failure of Hilbert-space factorization? Failure of classical spacetime itself?

An obstruction class in admissibility space would record the incompatibility of transition maps among policies. Suppose quantum descriptions agree with semiclassical descriptions on one overlap, and semiclassical descriptions agree with gravitational descriptions on another, but the composite comparison fails around a loop in Π . Then the obstruction is not local. Each pairwise chart may appear adequate, while global consistency fails. This is exactly the kind of phenomenon sheaf and cohomological methods are designed to detect.

The obstruction may also be scale-dependent. At low energy, quantum matter and classical geometry may glue approximately. At high curvature, the transition may fail. At macroscopic scales, decoherence and coarse-graining may stabilize spacetime. Near singularities or horizons, ordinary gluing may break. A policy-indexed formulation can represent this by allowing obstruction to appear in certain regions of Π and vanish in others.

Failed globalization changes the meaning of physical reality. It does not imply that there is no reality. It implies that reality is not captured by the attempted global presentation. The classical manifold may fail to globalize. The ordinary Hilbert-space factorization may fail to globalize. The external time parameter may fail to globalize. The local observable algebra may fail to globalize. The task is then to find a higher object whose local quotient presentations explain why these lower globalizations work where they work and fail where they fail.

This leads to the central mathematical proposal of the paper. Quantum gravity should be formulated as an admissibility sheaf or stack over policy space Π . Its local sections are quotient presentations of physical reality under admissible policies. Its restriction maps describe lawful change of policy. Its gluing conditions define compatibility. Its obstruction classes identify the

regimes where quantum and gravitational identity cannot be forced into a single lower presentation.

Section 6 has therefore converted the conceptual mismatch into a formal program. The base is policy space Π . The fibers are admissible physical description spaces. Local charts represent quantum, geometric, semiclassical, algebraic, holographic, observer-relative, or scale-dependent regimes. Restriction and transition maps express lawful comparison. Gluing expresses successful compatibility. Obstruction expresses failed globalization. The next section recasts this sheaf-theoretic picture categorically, where physical theories become functorial systems of admissible transformations rather than isolated equations.

7. Category Theory, Functorial Physics, and Admissible Transformation

The sheaf-theoretic formulation describes quantum gravity as a gluing problem over policy space. Category theory supplies the language of transformation needed to make that formulation structural rather than merely descriptive. Physics is not only about objects; it is about lawful relations among objects. States transform. Fields restrict. Observables compose. Measurements map preparations to outcomes. Coordinate charts transition. Gauge transformations identify presentations. Renormalization carries theories across scale. Dualities relate apparently different descriptions. Each of these is a map, and each map is admissible only when it preserves the identity that matters in context.

Category theory begins by refusing to treat objects in isolation. An object is understood through the morphisms into and out of it. This is already close to the operational and relational spirit of physics. A particle is known by interactions. A field is known by couplings and observables. A spacetime is known by causal and geometric relations. A quantum system is known by preparations, transformations, and measurements. A theory is known not only by what it contains, but by what transformations it permits.

Admissibility Algebra adds a further requirement. Not every morphism is physically admissible. A map must preserve the relevant invariants under the relevant policy. A coordinate transformation is admissible in general relativity when it respects diffeomorphic equivalence. A unitary transformation is admissible in closed quantum mechanics when it preserves inner products and probability structure. A gauge transformation is admissible when it preserves observable field content. A renormalization map is admissible when it preserves the correct low-energy or scale-relative physics. A duality is admissible when it preserves physical predictions across radically different presentations.

The categorical formulation therefore treats a physical theory as a structured system of admissible transformations. Objects are presentations or quotient objects. Morphisms are

lawful transformations. Functors relate regimes. Natural transformations compare functors. Stacks organize local quotient theories with gluing data. This section develops the categorical layer needed to support the sheaf formulation of quantum-GR gluing.

7.1 Categories of physical presentations

Let $\mathcal{P}$ be a category of physical presentations. Its objects may be quantum states, algebras of observables, field configurations, spacetime geometries, measurement contexts, boundary data, histories, or other structured descriptions. Its morphisms are transformations among these presentations: time evolutions, symmetry transformations, inclusions, restrictions, coarse-grainings, gauge maps, coordinate changes, measurement maps, or dynamical processes.

This category of presentations is not yet the physical theory. It contains too much. It may contain distinctions that are representational rather than physical. It may distinguish between coordinate descriptions that general relativity identifies, between gauge potentials that gauge theory identifies, between state vectors differing only by global phase, or between microstates that a thermodynamic policy intentionally forgets. Thus $\mathcal{P}$ is the raw or pre-quotient categorical domain.

An admissibility policy π acts on $\mathcal{P}$ by selecting equivalences and admissible morphisms. The result is a quotient or localized category $\mathcal{P}\pi$. In $\mathcal{P}\pi$, objects that differ only by π -irrelevant distinctions are identified, and morphisms that fail to preserve π -invariants are excluded or demoted. The physical category under π is therefore not $\mathcal{P}$ itself, but the policy-filtered structure:

$$\mathcal{P} \rightarrow \mathcal{P}\pi$$

The same raw presentation category may yield many physical categories under different policies. Under a quantum policy, $\mathcal{P}\pi$ may be a category of Hilbert spaces, states, operators, channels, or algebras. Under a gravitational policy, $\mathcal{P}\pi$ may be a category of Lorentzian manifolds, metric-field configurations, cobordisms, causal structures, or diffeomorphism classes. Under a thermodynamic policy, it may be a category of macrostates and entropy-preserving transitions. Under a holographic policy, it may relate boundary algebras and bulk geometries.

This multiplicity is not a weakness. It is the categorical expression of policy-indexed reality. The physical object is not merely an element of one category. It may be a compatible family of objects across categories. A black hole, for example, may appear as a classical metric object in one category, a thermodynamic object in another, a quantum information object in another, a boundary-state object in another, and a microscopic gravitational object in a still unknown category. The question is not which category is the only real one. The question is how these categories are connected by admissible functors and whether their quotient objects glue.

The category of presentations also clarifies why naive unification fails. If one attempts to place all quantum and gravitational objects into a single ordinary category too early, one must either impose one identity policy on all objects or allow inconsistent morphisms. A better approach is to keep policy-indexed categories and study the functors among them. Quantum-GR gluing is then not the collapse of all categories into one flat domain, but the construction of a higher categorical system in which local physical categories relate coherently.

7.2 Functors between quantum and geometric regimes

A functor maps one category to another while preserving compositional structure. In physics, a functor can express translation between regimes. It may send microscopic states to macroscopic states, quantum systems to classical limits, local algebras to spacetime regions, boundary data to bulk geometry, or geometric configurations to observable structures. A functor is not merely a dictionary. It preserves lawful relationships.

Let $\mathcal{Q}$ be a category of quantum descriptions under a policy π_Q . Let $\mathcal{G}$ be a category of gravitational descriptions under a policy π_{GR} . A quantum-geometric functor would have the form:

$$F: \mathcal{Q} \rightarrow \mathcal{G}$$

or, in the opposite direction:

$$G: \mathcal{G} \rightarrow \mathcal{Q}$$

A functor F from quantum to geometry might assign to a quantum state or algebra an emergent metric, causal structure, entropy relation, or geometric quotient. A functor G from geometry to quantum might assign to a spacetime region an algebra of fields, a Hilbert space, a path integral, or a quantum state space. Neither direction should be assumed fundamental in advance.

Different regimes may require different directions of construction.

For a functor to be admissible, it must preserve the invariants relevant to the target policy while respecting the equivalences of the source policy. If two quantum presentations are equivalent under π_Q , their images must be equivalent under the appropriate geometric policy, unless the functor explicitly records an obstruction. Similarly, if two geometric presentations are diffeomorphically equivalent, their quantum images must not depend on arbitrary coordinate artifacts. This is the categorical expression of admissibility preservation.

The difficulty of quantum gravity can be stated functorially. There are many partial functors between quantum and geometric regimes, but no universally accepted functorial system that preserves all required invariants. Quantum field theory on curved spacetime provides a map from geometric backgrounds to quantum field algebras. Semiclassical gravity provides a map from quantum stress-energy expectations to approximate geometry. Effective quantum gravity

provides perturbative quantum corrections around a background. Holography suggests a deeper relation between boundary quantum theories and bulk gravitational descriptions. Each is a partial functor. None is yet the full admissibility-covariant functor.

The functorial viewpoint prevents a common simplification. It is not enough to say that spacetime emerges from entanglement, or that quantum theory emerges from geometry, or that one side is more fundamental. One must specify the functor, its domain, its codomain, its preserved invariants, its treatment of equivalence, its behavior under composition, and its failure modes. A slogan is not a functor. A proposed emergence relation becomes mathematical only when it gives lawful transport between categories.

A valid quantum-GR functor should also respect scale. At low energy, it should reproduce classical gravitational behavior. At ordinary laboratory scales, it should reproduce standard quantum predictions. In semiclassical regimes, it should explain approximate compatibility. At high curvature or near horizons, it should identify whether gluing persists or obstruction appears. Thus the functor is not merely algebraic. It is policy-indexed, scale-sensitive, and obstruction-aware.

7.3 Natural transformations as admissible comparison maps

Often physics supplies not one translation between regimes but several. Different constructions may map quantum data to geometric data. Different choices of gauge, regularization, foliation, renormalization scheme, or boundary condition may yield apparently different functors. The question is whether these functors are physically equivalent. Category theory answers this through natural transformations.

Suppose F and H are two functors from a category $\mathcal{A}$ to a category $\mathcal{B}$:

$$F, H: \mathcal{A} \rightarrow \mathcal{B}$$

A natural transformation $\eta: F \Rightarrow H$ assigns to each object A in $\mathcal{A}$ a morphism $\eta_A: F(A) \rightarrow H(A)$ in $\mathcal{B}$, compatible with every morphism in $\mathcal{A}$. In physical language, a natural transformation is a coherent comparison between two ways of translating a system. It says not merely that two outputs are individually comparable, but that the comparison respects all admissible processes.

This matters for quantum gravity because many proposed descriptions may be locally equivalent without being globally or naturally equivalent. Two quantization procedures may agree on some observables but not on composition. Two gauge choices may yield the same local predictions but differ in global structure. Two semiclassical approximations may match in one regime but fail under transition to another. A natural transformation tests whether equivalence is coherent across the whole category of processes.

Admissibility Algebra strengthens this. A natural transformation is physically meaningful only if its components are admissible morphisms under the relevant policies. It must preserve the invariants that define identity in the target category. If η_A identifies structures that differ in a measurable or invariant way, it is not admissible. If it depends on arbitrary presentation choices forbidden by the source or target policy, it is not admissible. If it works object-by-object but fails to commute with transformations, it is not natural and therefore not a lawful comparison of theories.

Natural transformations also express changes of presentation inside one theory. A change of coordinates in general relativity, a gauge transformation in field theory, a change of basis in quantum mechanics, or a change of renormalization scheme may be represented as a natural equivalence when it coherently preserves physical content. The point is not that all such transformations are identical, but that they share the same structural role: they compare descriptions without changing admissible identity.

In the quantum-GR setting, one may need natural transformations between functors such as a semiclassical functor and a holographic functor, or between a canonical quantization functor and a path-integral functor, or between a boundary reconstruction functor and a bulk effective field functor. The existence or nonexistence of admissible natural transformations among these functors becomes evidence for or against genuine gluing.

A gluing failure may therefore appear as failure of naturality. The local objects match, but the morphisms do not. Predictions agree for isolated states, but not under evolution. Geometries match under one observer policy, but not under another. Boundary data reconstructs a bulk region, but not in a way compatible with all causal or entanglement transformations. Such failures are not merely technical. They reveal that the proposed comparison does not preserve admissible identity.

Natural transformations are therefore the categorical version of lawful comparison. A quantum-GR theory must not only assign quantum objects to geometric objects. It must compare different such assignments coherently. Where coherent comparison exists, gluing becomes plausible. Where it fails, obstruction becomes visible.

7.4 Stacks of quotient theories

A sheaf assigns local data to regions and glues them when they agree. A stack generalizes this by allowing the local data themselves to have internal symmetries, equivalences, and higher morphisms. This is essential for physics because physical descriptions are rarely sets of bare objects. They are categories or higher categories of objects related by gauge transformations, diffeomorphisms, dualities, and equivalences. A stack is therefore a natural home for quotient theories.

A quotient by symmetry often cannot be adequately represented by a simple set of equivalence classes. The equivalence itself carries structure. Gauge fields, bundles, moduli spaces, and diffeomorphic geometries often require keeping track not only of objects but of their automorphisms and transition data. If one collapses too aggressively to a set, one loses information. A stack preserves the quotient while retaining the symmetry structure responsible for it.

In Admissibility Algebra, this is crucial. A policy π does not merely produce $[X]_\pi$ as a flat equivalence class. It may produce a structured quotient with residual symmetries, higher identifications, obstruction data, and transformation laws. Thus the fiber over π should often be a category, groupoid, or higher groupoid, not a set. The global object over Π should therefore be a stack of admissible quotient theories.

A stack of quotient theories assigns to each policy region U a category $\mathcal{S}(U)$ of quotient presentations valid over U . Objects of $\mathcal{S}(U)$ are local physical descriptions. Morphisms are admissible transformations between them. Higher morphisms may represent equivalences between transformations, gauge-of-gauge structure, homotopies, or coherent identity witnesses. Restriction maps carry these categories across policy refinement or localization. Descent conditions specify when local quotient categories glue into a global quotient category.

Quantum gravity appears to require stack-like structure because both quantum theory and general relativity carry nontrivial equivalence data. Quantum theory has phase equivalence, unitary equivalence, algebraic equivalence, dual descriptions, and contextual measurement relations. General relativity has diffeomorphism equivalence, local frame transformations, constraint structure, and relational observables. Holography adds dual equivalences between apparently different dimensional and geometric presentations. These cannot be faithfully represented by a single set of objects.

A stack formulation also handles automorphisms. Some physical objects have nontrivial self-equivalences: symmetries that preserve the object. These symmetries are physical data. A highly symmetric spacetime, a gauge field with stabilizer group, a quantum state invariant under a symmetry, or a boundary theory with dual reconstructions contains information in its automorphism structure. Quotienting should not erase this structure; it should organize it.

Thus quantum gravity should not be formulated merely as a sheaf of states or a set of geometries. It should be formulated as a stack of policy-indexed quotient categories. The local categories encode quantum, geometric, semiclassical, and boundary descriptions. The descent data encode how they glue. The obstructions encode where they fail. The higher morphisms encode identity above equality.

This explains why many quantum gravity programs feel close but incomplete. Each may capture a chart or substack: canonical constraints, spin networks, causal sets, string backgrounds, holographic dualities, effective theories, algebraic nets, or semiclassical approximations. But the full theory would require the stack in which these charts are recognized as admissible local quotient theories and compared by coherent transition data.

7.5 Physics as lawful transport across admissibility policies

The categorical and stack-theoretic picture leads to a broader definition of physics. Physics is the study of lawful transport across admissibility policies. A law of physics does not merely describe how an object changes in time. It describes what remains invariant, how descriptions transform, which substitutions are permitted, and how quotient presentations persist across changes of context, scale, observer, gauge, and regime.

Ordinary covariance is one instance of lawful transport. It says that physical content survives coordinate change. Gauge invariance is another. It says that observable content survives gauge transformation. Unitary evolution is another. It says that quantum probability structure survives closed-system time development. Renormalization is another. It says that effective physical content transforms across scale. Duality is another. It says that apparently different theories may be equivalent presentations of the same physics. Admissibility Algebra unifies these as forms of policy transport.

Quantum gravity requires the highest form of such transport because the policies themselves become mutually entangled. Measurement policy depends on causal structure. Causal structure depends on geometry. Geometry is dynamical. Quantum states depend on observable algebras. Observable algebras depend on locality. Locality may be emergent or holographic. Thus one cannot hold all policies fixed while transforming objects. The policies transform too.

A fully admissibility-covariant physical theory should therefore specify:

objects over policies,
morphisms within each policy,
functors between policy regions,
natural transformations comparing functors,
descent data for gluing,
obstruction classes for failed globalization,
and invariants preserved under admissible transport.

This is not abstraction for its own sake. It is the minimum structure required when physical identity is not absolute. If a theory claims that two descriptions are equivalent, it must say by what functor. If it claims that two functors are equivalent, it must say by what natural transformation. If it claims that local descriptions form one global object, it must provide descent data. If it claims that gluing fails, it must identify the obstruction.

The categorical interpretation of Admissibility Algebra therefore changes the aim of quantum gravity. The goal is not merely to choose the right primitive objects. The goal is to construct the right transformation system. Whether the primitives are strings, loops, spin networks, algebras, causal sets, categories, amplitudes, or histories is secondary to whether their admissibility policies transport correctly. A primitive without lawful transport is just another presentation.

In this view, quantum theory and general relativity are two highly successful local categories of physical identity. Quantum theory excels in the transport of amplitudes, operators, probabilities, and contextual correlations. General relativity excels in the transport of geometry, causal structure, covariance, and diffeomorphic identity. Quantum gravity should be the admissibility-covariant stack in which these transports become compatible where possible and explicitly obstructed where not.

Section 7 has therefore recast the sheaf formulation categorically. Physical presentations form categories. Policies produce quotient categories. Regime changes are functors. Comparisons of regime changes are natural transformations. Full quotient structures form stacks. Physics becomes lawful transport across admissibility policies. The next section will move from category theory to homotopy type theory, where the identity layer itself can be treated as structured, higher-dimensional, and internal to the formal system.

8. Homotopy Type Theory and Identity Above Equality

Homotopy type theory enters this program because the quantum-GR obstruction is not merely a problem about objects. It is a problem about identity. Ordinary equality is too thin for modern mathematical physics. Gauge equivalence, diffeomorphism invariance, duality, contextual equivalence, categorical equivalence, homotopy, and operational indistinguishability all show that physics often depends on structured sameness rather than literal equality. Homotopy type theory provides a formal environment in which identity may carry structure, paths may represent equivalences, and higher paths may compare equivalences among equivalences. This makes it a natural partner for Admissibility Algebra.

In ordinary set-based thinking, two objects are either equal or not equal. But physical theories rarely operate with such a simple distinction. Two coordinate presentations of the same spacetime may not be literally identical, but they may be physically equivalent. Two gauge

potentials may differ, yet represent the same observable field. Two quantum state-vector representatives may differ by global phase, yet represent the same physical ray. Two dual theories may have different variables, dimensions, or geometric interpretations, yet produce the same physical content. These are not weak forms of equality. They are structured identities.

Homotopy type theory treats identity as path-like. To say that x equals y is not merely to assert a static proposition. It is to provide an identity witness, a path from x to y . There may be many such paths. Paths themselves may be compared by higher paths. This hierarchy is well suited to physics, where equivalence often has internal structure: one gauge transformation may differ from another, one coordinate transition may compose with another, one duality may interact with another, and one admissibility policy may be refined or transported into another.

Admissibility Algebra contributes the policy layer. Homotopy type theory explains how identity can be structured; Admissibility Algebra asks which structured identities are admissible in a given physical context. Together they suggest that quantum gravity may require not merely a new space of states, but a higher identity system in which quantum and geometric equivalences are themselves objects of formal comparison.

8.1 Equality, equivalence, and univalence

The distinction between equality and equivalence is central. Equality, in its strict form, says that two expressions are the same within a given formal system. Equivalence says that two structures may differ in presentation while preserving the organization that matters. In mathematics, equivalence is often more important than literal equality. Isomorphic groups, homeomorphic spaces, equivalent categories, homotopy-equivalent spaces, and gauge-equivalent fields are not literally the same as presentations, yet they may be the same for the relevant theory.

Univalence sharpens this principle. It states, roughly, that equivalence can itself be treated as identity, provided the equivalence is of the appropriate kind. This does not mean careless identification. It means that when two structures are equivalent in the right way, the theory may transport properties between them as if they are identical. Identity becomes structured by admissible equivalence.

This is already the spirit of mathematical physics. A coordinate transformation in general relativity does not merely relate two different descriptions; it licenses treating them as the same physical geometry. A gauge transformation does not merely map one potential to another; it licenses treating them as the same gauge field when the observable content is preserved. A duality does not merely compare two theories externally; in strong cases it says that the two theories are different presentations of the same physical content.

Admissibility Algebra inserts the missing question: equivalence under which policy? Not every equivalence is physically admissible. A map that preserves one invariant may destroy another. A coarse-graining may be valid thermodynamically but invalid for quantum interference. A coordinate transformation may be admissible gravitationally but irrelevant to Hilbert-space factorization. A unitary equivalence may preserve quantum probability while failing to respect a desired geometric interpretation. Therefore univalence must be policy-indexed for physics.

One may write the guiding idea as:

equivalence under π may be treated as identity under π .

This is the admissibility-covariant reading of univalence. Identity is not flattened into arbitrary sameness. It is licensed by a policy π that specifies the equivalence relation, preserved invariants, and permitted transport. The quotient $[X]\pi$ is therefore not only a set-theoretic equivalence class. It is a type of admissible presentations connected by identity paths sanctioned by π .

For quantum gravity, this matters because quantum and gravitational equivalences are different. Quantum equivalence may concern rays, unitary maps, operator algebras, density matrices, or operational statistics. Gravitational equivalence may concern diffeomorphism classes, causal structure, curvature relations, or relational observables. A final theory must say when these different equivalences are themselves equivalent, when they can be transported into one another, and when they cannot.

8.2 Physical identity as structured equivalence

Physical identity is not bare equality. It is structured equivalence constrained by invariants. Two descriptions count as the same physical object when there exists an admissible path between them preserving the identity-relevant content. This path may be a gauge transformation, coordinate transformation, unitary map, duality, homotopy, categorical equivalence, measurement-preserving operational equivalence, or scale transition. The nature of the path depends on the policy.

Homotopy type theory allows this to be stated internally. Instead of saying from outside the theory that X and Y are equivalent, one may represent an identity witness $p: X = Y$. This witness has content. It is not merely a yes-or-no label. The path p may itself be transformed, compared, composed, inverted, restricted, or lifted. In physics, this corresponds to the fact that transformations of identity are themselves physically meaningful.

For example, suppose two gravitational presentations G and G' are related by a diffeomorphism. The diffeomorphism is an identity witness under the gravitational policy. It says $G \equiv_{\pi_{GR}} G'$. But different diffeomorphisms may relate the same presentations, and those differences may matter

in the presence of boundaries, topology, asymptotic symmetries, or global structure. Thus the identity witness has structure. A set-level quotient may erase too much. A higher identity theory preserves the organization of equivalences.

Similarly, two quantum presentations ψ and $e^{i\theta}\psi$ are equivalent under global phase. The phase transformation is an identity witness under a suitable quantum policy. But phase cannot be globally dismissed in all contexts. Relative phase, Berry phase, holonomy, interference, and gauge coupling may make phase structure observable. Again, the identity relation is not simply “same” or “different.” It is structured by policy, context, and higher relations.

Physical identity as structured equivalence allows a more refined realism. A physical object is not a raw presentation; neither is it a featureless equivalence class. It is a structured quotient object whose internal paths encode admissible transformations. This is crucial in gauge theory, general relativity, and quantum theory, where the transformations preserving identity often carry physical information.

The quantum-GR problem can then be restated. We possess structured equivalences for quantum theory and structured equivalences for general relativity, but we do not yet possess a higher system that identifies when quantum identity paths and gravitational identity paths are compatible. A quantum-gravitational object must carry paths corresponding to quantum admissibility, paths corresponding to diffeomorphic admissibility, and higher paths comparing these identity systems where they overlap.

This is why homotopy type theory is not an ornament. It gives the formal capacity to discuss identity above equality. Quantum gravity appears to require exactly that: not merely objects and equations, but a higher-dimensional identity architecture.

8.3 Paths, higher paths, and gauge-like identity

In homotopy type theory, an identity between x and y may be represented as a path. An identity between two such paths may be represented as a higher path. This hierarchy continues. The resulting structure resembles the way gauge theories, higher gauge theories, and geometric field theories organize transformations, transformations between transformations, and coherence conditions.

Gauge identity is naturally path-like. If A and A' are gauge-related presentations, a gauge transformation provides a path between them in the space of admissible descriptions. If two gauge transformations differ by a higher gauge transformation, there may be a higher path between paths. In theories with constraints, bundles, gerbes, higher connections, or extended objects, such higher structure is not optional. It is part of the geometry of the theory.

General relativity also has path-like identity. Diffeomorphisms relate presentations of spacetime geometry. Compositions of diffeomorphisms must be coherent. Local charts glue through transition maps. Boundaries and asymptotic regions can turn some transformations from pure redundancy into physical symmetries. The identity structure is therefore not merely a quotient by a group action; it may involve higher categorical and homotopical data.

Quantum theory has its own path-like identity. Continuous unitary evolution traces paths in projective Hilbert space. Gauge phases may produce holonomies. Adiabatic transport can generate geometric phases. Measurement contexts may be organized into compatibility structures whose failure to globalize has sheaf-like form. Entanglement transformations and local unitary equivalences define structured pathways through state space. The quantum object is not a static point but a network of admissible transformations.

The phrase “gauge-like identity” names this general phenomenon. Identity is not a primitive equality relation imposed from above. It is generated by admissible transformations. Two presentations are the same when there is a permitted path connecting them. Two ways of connecting them may themselves be compared. The theory must specify which paths exist, how they compose, which paths are equivalent, and what invariants they preserve.

This gives a natural interpretation of policy-indexed quotients. A policy π determines a path structure on presentations. The quotient $[X]\pi$ is the object X together with all π -admissible identity paths and higher paths. If π is coarse, more paths are admitted and more distinctions are identified. If π is fine, fewer paths are admitted and more distinctions survive. If π and ρ admit incompatible path structures, their quotients may fail to glue.

In quantum gravity, the obstruction may be a failure of higher path coherence. Quantum paths of admissible identity and gravitational paths of admissible identity may agree locally but fail around loops in policy space. A transformation that is quantum-admissible may not be gravitationally admissible. A diffeomorphic identification may not preserve a quantum factorization or measurement context. A unitary map may not respect relational geometry. The higher coherence conditions fail.

Such failure should not be treated merely as inconsistency. It may be the mathematical signal of quantum-gravitational structure. A singularity, horizon, anomaly, contextuality obstruction, or failure of global time may correspond to a place where identity paths cannot be coherently completed. The higher identity structure does not erase these problems; it makes them visible.

8.4 Quantum-GR gluing as higher identity construction

If quantum gravity is a gluing problem, then the object being glued is not merely a collection of states or geometries. It is a collection of identity systems. Quantum theory supplies one higher

identity system; general relativity supplies another. The sought theory must construct a higher identity environment in which both can appear as local or quotient identities.

Let X be a deeper physical presentation. Under a quantum policy π_Q , X yields a quotient $[X]_{\pi_Q}$ with quantum identity paths: phase equivalences, unitary transformations, algebraic equivalences, contextual measurement relations, and entanglement-preserving maps. Under a gravitational policy π_{GR} , X yields a quotient $[X]_{\pi_{GR}}$ with geometric identity paths: diffeomorphisms, frame transformations, causal-relational identifications, and background-independent equivalences. The gluing problem asks whether these quotient identity systems are compatible over an overlap policy π_{QG} .

Compatibility requires more than matching objects. It requires matching identity witnesses. Suppose a quantum description and a gravitational description agree on some invariant, such as energy, entropy, causal relation, or boundary correlation. That agreement is insufficient if the transformations preserving the invariant do not also agree. A true glue must preserve the way identity is transported.

This is analogous to gluing bundles rather than gluing pointwise values. One does not merely check that local fields assign the same numbers on overlaps. One checks transition functions and cocycle conditions. In quantum-GR gluing, one must check transition functors, natural transformations, higher paths, and coherence laws. The local quantum and geometric descriptions must descend from a single higher object only if their identity structures agree on overlaps.

A higher identity construction may have several levels:

objects: local quantum or geometric quotient presentations,

paths: admissible transformations within each policy,

two-paths: equivalences between transformations,

higher paths: coherence among equivalences,

descent data: rules for gluing across policy overlaps,

obstruction data: failures of coherence.

This hierarchy is exactly what homotopy type theory and higher category theory are designed to formalize. It also matches the physical intuition that quantum gravity is not merely about replacing a metric by an operator. It is about replacing absolute identity by a structured system of admissible equivalences.

Classical spacetime can then be interpreted as a lower truncation of the higher identity structure. At macroscopic scales, many higher distinctions become invisible or irrelevant. The quotient stabilizes. The manifold, metric, and causal structure appear as if they possessed ordinary object identity. But this classical identity is a successful gluing product, not a primitive fact. It is the result of higher identity data becoming coherent under coarse admissibility policies.

Quantum ambiguity is the opposite case. Higher identity data refuse to collapse into a single classical section. Noncommuting contexts, entangled subsystems, and measurement-dependent outcomes indicate that no policy-independent identity object exists at the classical level. Quantum gravity may involve similar refusal at the level of spacetime. There may be no single geometry before policy, just as there is no single set of sharp values before measurement context.

The quantum-GR glue, therefore, is a higher identity construction whose ordinary quantum and classical geometric theories are shadows. Quantum theory is the algebraic-contextual shadow. General relativity is the diffeomorphic-geometric shadow. Quantum gravity is the higher object whose admissibility-indexed projections generate both.

8.5 Why HoTT clarifies the identity layer beneath physics

Homotopy type theory clarifies the identity layer because it refuses to reduce identity to a single flat relation. It permits identity to be inhabited by paths, structured by equivalences, layered by higher comparisons, and disciplined by transport. This is precisely what modern physics requires. The most important identities in physics are not literal equalities of notation. They are equivalences that preserve physical content under lawful transformation.

Admissibility Algebra tells us that such equivalences must be policy-relative. Homotopy type theory tells us that they can be internal, structured, and higher-dimensional. Together they provide a language for physical identity above equality. This is not a replacement for experiment, dynamics, or calculation. It is the formal grammar needed before equations can be interpreted correctly across incompatible regimes.

The quantum-GR problem exposes the need for this grammar. If one treats quantum state identity as absolute, one risks violating background independence. If one treats diffeomorphic geometric identity as absolute, one risks losing quantum contextuality, measurement structure, and unitarity. If one treats all equivalences as informal analogies, one cannot distinguish true gluing from superficial resemblance. Homotopy type theory supplies the formal discipline for equivalence; Admissibility Algebra supplies the policy discipline for admissibility.

This also explains why univalence matters. In a univalent setting, equivalent structures may be identified when the equivalence is appropriate. Physics has long practiced this informally. It

treats gauge-equivalent fields as the same, coordinate-equivalent geometries as the same, and dual theories as potentially the same. But quantum gravity requires a more exact version: equivalent under what admissibility policy, preserving which invariants, with what higher coherence, and over which region of policy space?

HoTT also makes transport central. If P is a property or structure associated with X , and p is an admissible identity path from X to Y , then one may transport P along p to obtain corresponding structure on Y . This is the formal analogue of physical covariance. Laws, observables, and invariants should transport along admissible identity paths. If transport fails, the alleged identity was not physically admissible for that structure.

This gives a powerful test for quantum-GR proposals. A candidate equivalence between quantum and geometric descriptions must support transport of the relevant structures. Can probabilities transport? Can causal relations transport? Can entropy transport? Can observables transport? Can locality transport? Can gauge and diffeomorphic equivalences transport? Can measurement records transport? If not, the proposal may be an analogy rather than a glue.

The identity layer beneath physics is therefore not metaphysical decoration. It is operational, mathematical, and structural. It determines what the theory means by same state, same geometry, same observable, same event, same process, and same law. Quantum gravity cannot avoid this layer because the theories being glued define these notions differently.

Section 8 has argued that homotopy type theory provides the correct formal sensitivity for identity above equality. Equality becomes path-like. Equivalence becomes identity when admissible. Higher paths encode coherence among transformations. Policy-indexing determines which equivalences are lawful. Quantum-GR gluing becomes the construction of a higher identity system whose local quotient presentations appear as quantum theory and general relativity. The next section will examine how decoherence and classical emergence can be reinterpreted as stabilization of such quotient identities across broad families of admissibility policies.

9. Decoherence, Classical Emergence, and Stabilized Quotients

Decoherence is usually described as the process by which quantum superpositions become effectively classical through entanglement with an environment. Interference terms become inaccessible, pointer states stabilize, and macroscopic records acquire robust persistence. This account is physically powerful, but Admissibility Algebra reveals a deeper identity structure. Decoherence is not merely loss of phase information. It is the stabilization of quotient presentations across a large family of admissibility policies.

A quantum system by itself may support many incompatible descriptions. Different measurement policies expose different invariants. Different bases preserve different distinctions. Different subsystem decompositions produce different reduced states. Quantum ambiguity arises because these quotient presentations do not generally glue into one classical object. Decoherence changes the situation by coupling the system to many environmental degrees of freedom. The environment does not merely observe the system in a human sense. It repeatedly selects, amplifies, and stabilizes certain distinctions while suppressing others. A small set of quotient presentations becomes robust across many policies.

This is why classicality appears objective. A classical object is not classical because it possesses absolute identity independent of all policy. It is classical because many admissibility policies agree on the same quotient identity. Many observers, apparatuses, environmental interactions, coarse-grainings, and scales recover approximately the same object. The object can be reidentified. Its records persist. Its position, shape, trajectory, and macroscopic properties survive lawful transformation across contexts. The indexing by policy becomes invisible because the quotient has stabilized.

The same logic may apply to spacetime. Classical spacetime may not be a primitive arena but a stabilized quotient emerging when many quantum, geometric, environmental, and observational policies agree. At ordinary scales, the metric, causal order, locality, and temporal succession appear robust. At quantum-gravitational scales, those quotients may destabilize. Decoherence and classical emergence therefore become central to quantum-GR gluing: they explain how policy-relative quantum and geometric descriptions may become stable enough to appear as one classical world.

9.1 Decoherence as multi-policy stabilization

Decoherence begins when a system becomes entangled with environmental degrees of freedom in such a way that certain interference relations become practically inaccessible. In the ordinary account, the reduced density matrix of the system becomes approximately diagonal in a preferred basis. The environment carries away phase information. The system appears to occupy one of several stable alternatives rather than a coherent superposition. This explains why macroscopic objects do not ordinarily display visible quantum interference.

In admissibility language, the environment acts as a large family of policies. Each interaction with the environment selects distinctions that can be amplified and recorded. Distinctions that cannot survive environmental coupling are suppressed or dispersed into inaccessible correlations. The system presentation X is repeatedly quotiented under many policies $\pi_1, \pi_2, \pi_3, \dots$ corresponding to environmental interactions, measurement channels, thermal couplings, scattering events, and coarse-grained observational access.

If many policies produce compatible quotient objects, then a stable classical identity appears:

$$[X]\pi_1 \approx [X]\pi_2 \approx [X]\pi_3 \approx \dots$$

The approximation is important. Classicality is rarely exact. It is robust enough for a domain of practice. The quotients agree to the level of resolution relevant for macroscopic prediction, record formation, and observer reidentification. Quantum coherence may remain in the global state, but it is no longer available under the local policies that define practical physical identity.

This reframes decoherence as multi-policy stabilization. The environment does not merely destroy quantum information. It redistributes distinctions across policy space. Some distinctions become inaccessible locally but remain present in a larger entangled presentation. Others become amplified into records. Still others become irrelevant under coarse-graining. The classical object is the quotient that survives this repeated filtering.

This also explains why pointer states are privileged. They are not privileged by metaphysical decree. They are stable under environmental admissibility. A pointer state is a quotient presentation that can be redundantly encoded, reidentified, and compared across many interactions. It is an attractor in admissibility space. It survives the policies that the environment imposes.

Quantum ambiguity is therefore not removed absolutely. It is localized into inaccessible or nonclassical parts of the total structure. The local classical quotient becomes stable while the full quantum presentation remains richer. Decoherence does not prove that only one outcome exists in every interpretation; rather, it explains why many policy-indexed descriptions converge on the same effective record. For the purposes of quantum-GR gluing, this is enough to reveal a general principle: classical identity appears when quotient identities become redundant, stable, and transportable across policies.

9.2 Classical spacetime as a robust quotient

Classical spacetime may be understood in the same way. General relativity treats spacetime as a smooth Lorentzian geometry modulo diffeomorphism, but this may be an effective quotient rather than a primitive structure. At macroscopic scales, many physical procedures agree on approximately the same spacetime: clocks measure stable proper times, rulers measure stable distances, light signals define stable causal cones, massive bodies trace stable geodesic behavior, and curvature produces reproducible tidal effects. These agreements make spacetime appear objectively present.

In Admissibility Algebra, classical spacetime is a robust quotient $[X]\pi_{\text{geo}}$ of a deeper presentation X . The policy π_{geo} preserves causal order, metric relations, curvature, local inertial structure, and diffeomorphism-invariant geometry while discarding microscopic or

quantum-gravitational distinctions that do not affect the classical regime. The stability of spacetime comes from the broad compatibility of many policies: observer frames, coordinate charts, matter probes, electromagnetic signals, clock comparisons, gravitational measurements, and coarse-grained energy distributions.

The smooth manifold is therefore not necessarily the deepest reality. It is a successful gluing product. Local coordinate charts glue into a manifold. Local inertial frames glue into curved geometry. Local measurements glue into causal and metric relations. Macroscopic matter distributions glue into stress-energy descriptions. Under ordinary conditions, these quotient presentations are compatible enough to support a single classical spacetime model.

This viewpoint changes the meaning of singularities and Planck-scale breakdown. A singularity need not be interpreted simply as a place where curvature becomes infinite in an otherwise fundamental continuum. It may indicate failure of the classical spacetime quotient. The policies that normally stabilize geometry no longer agree. Proper time, curvature, causal extension, quantum stress-energy, horizon information, and microscopic degrees of freedom may fail to glue into one smooth geometric object.

Similarly, quantum fluctuations of geometry may be interpreted as instability of the classical quotient. If causal order, distance, or metric relations cannot be stabilized across policies, then the classical spacetime object fails to exist at that resolution. This does not mean nothing exists. It means the admissible quotient called classical spacetime is no longer the right object. One must seek a deeper presentation whose other quotient projections remain lawful.

The robustness of classical spacetime also explains why general relativity is so successful. The quotient $[X]\pi_{\text{geo}}$ glues over a vast range of scales and conditions. It is not a crude illusion. It is a stable emergent invariant. A theory of quantum gravity should not dismiss it. It should explain why this quotient is so robust, where it comes from, which policies stabilize it, and why it fails only in extreme regimes.

9.3 Macroscopic objects as glued quotient families

Macroscopic objects are not raw microscopic configurations. They are glued quotient families. A chair, rock, planet, organism, instrument, or laboratory is composed of vast numbers of microscopic degrees of freedom, most of which are irrelevant to its macroscopic identity. The object persists because many admissibility policies recover the same coarse structure despite microscopic change.

A chair remains the same chair while its molecules vibrate, exchange heat, absorb photons, shed particles, and undergo minute chemical changes. A planet remains the same planet while its atmosphere circulates, crust shifts, radiation escapes, and matter accretes or erodes. A biological organism remains the same organism through cellular turnover, metabolism, repair,

and environmental exchange. These identities are not literal equality of microstate. They are policy-stabilized quotient identities.

For a macroscopic object O , many policies π_i produce compatible quotients:

$[O]\pi_{\text{visual}}, [O]\pi_{\text{mechanical}}, [O]\pi_{\text{thermal}}, [O]\pi_{\text{gravitational}}, [O]\pi_{\text{social}}, [O]\pi_{\text{instrumental}}$

These quotient presentations are not identical in content. The visual policy preserves shape and color. The mechanical policy preserves rigidity and load-bearing structure. The thermal policy preserves temperature and heat capacity. The gravitational policy preserves mass-energy distribution. The instrumental policy preserves calibration and functional response. Yet these policies glue sufficiently to support one reidentifiable object.

This is the reason macroscopic identity feels natural. It is not because every microscopic detail is fixed. It is because the quotient family is coherent. Different observers and instruments can compare records, repeat measurements, and act on the object without renegotiating its identity at every moment. Its objecthood is a gluing achievement.

The same logic applies to measurement apparatus. A detector is classical enough to record quantum outcomes because its relevant states are stabilized across environmental and operational policies. Pointer positions, electronic signals, memory states, and printed records persist through redundant encoding. The apparatus is not outside quantum theory in principle. It functions classically because its quotient identity is robust.

In quantum gravity, the macroscopic apparatus cannot be ignored. Measurement requires records, and records require stabilized quotient identities. But the apparatus is also gravitational. It has mass, worldline, causal access, and spacetime localization. A quantum-gravitational account of measurement must therefore explain how macroscopic glued quotient families arise within a dynamical spacetime setting. The detector, observer, and record are not external primitives. They are emergent stabilized quotients within the same admissibility-covariant structure.

Macroscopic objects also teach caution. Stability is domain-relative. An object may be stable under ordinary policies and fail under extreme policies. A table is stable mechanically but not under combustion. A planet is stable gravitationally but not under catastrophic collision. A spacetime region is stable classically but perhaps not under Planck-scale probing. Thus objecthood is not absolute; it is robust over a region of policy space. Quantum gravity must map those regions.

9.4 Environmental policies and admissible agreement

The environment is often described as a reservoir, bath, or collection of uncontrolled degrees of freedom. In the admissibility framework, the environment is also a policy generator. Every

environmental interaction tests, selects, records, or erases distinctions. Photons scatter from surfaces. Air molecules collide with objects. Thermal fields exchange energy. Gravitational interactions correlate masses. Cosmic background radiation carries traces. Matter fields couple to geometry. These interactions impose admissibility pressures on physical presentations.

An environmental policy need not be intentional. It is a physical process that determines which distinctions can survive interaction and become redundantly encoded. When many environmental channels select the same distinction, that distinction becomes classical. When they disperse a distinction into inaccessible correlations, that distinction becomes locally irrelevant. The environment thus constructs a practical quotient by repeated selection.

Admissible agreement occurs when independent policies converge on the same invariant. For example, the position of a macroscopic object may be inferred by sight, touch, radar reflection, acoustic echo, gravitational influence, and mechanical contact. These are distinct policies. Their agreement stabilizes the object's location. If they disagree radically, object identity becomes problematic. A mirage, quantum interference experiment, gravitational lensing ambiguity, or horizon-separated description reveals that policy agreement is not automatic.

The strength of classical reality lies in redundant admissible agreement. Many policies independently recover the same quotient. This is closely related to the idea that classical information becomes objective when it is redundantly recorded in the environment. Admissibility Algebra generalizes that idea: objectivity is broad gluing across policy space.

For spacetime, environmental policies include matter propagation, light cones, clocks, rods, thermodynamic arrows, causal accessibility, gravitational waves, quantum field correlations, and horizon structure. Classical geometry is stabilized when these policies agree on a common metric-causal quotient. At extreme scales, they may not. Quantum matter may not define a sharp stress-energy tensor. Causal order may fluctuate. Horizon information may produce incompatible observer descriptions. Then environmental agreement weakens or fails.

This interpretation allows a precise distinction between subjectivity and policy-relativity. A policy-relative object is not subjective if the policy is physically instantiated and publicly reproducible. A clock policy, scattering policy, interferometer policy, or diffeomorphic policy is constrained by physical law. The objectivity of a quotient is measured by the range of admissible policies under which it remains stable. The more widely it glues, the more objective it is.

Quantum gravity should therefore not ask whether spacetime is real or emergent in a vague sense. It should ask: across which environmental and measurement policies does the spacetime quotient stabilize? Which invariants are redundantly agreed upon? Which fail near horizons, singularities, or Planck regimes? Which deeper quotient replaces classical geometry when agreement breaks down? These are admissibility questions.

9.5 Emergence without reducing geometry to measurement alone

The admissibility view risks misunderstanding if stated too quickly. To say that classical geometry emerges as a stabilized quotient is not to say that geometry is merely created by measurement, consciousness, language, or convention. Nor does it mean that spacetime is nothing but an observer's bookkeeping device. The point is subtler: geometry is real as an invariant quotient stabilized across lawful physical policies.

Measurement is only one kind of policy. Gauge equivalence, diffeomorphism invariance, coarse-graining, environmental coupling, scale restriction, causal access, and dynamical stability are also policies. Classical spacetime emerges not because observers decide to see it, but because many independent physical processes agree on the same geometric quotient. Its emergence is objective precisely because the quotient is robust.

This avoids two extremes. The first extreme is naive geometric absolutism, which treats smooth spacetime as the primitive container of all physical reality. Quantum gravity challenges this because the smooth manifold and metric may fail at fundamental scales. The second extreme is naive measurement idealism, which treats geometry as merely an artifact of observation. General relativity and classical physics reject this because spacetime geometry has invariant structure, constrains motion, carries gravitational degrees of freedom, and produces observer-independent relational effects. The admissibility view occupies a third position.

On this third view, geometry is neither primitive nor unreal. It is emergent, invariant, and policy-stabilized. It is real in the same disciplined sense that temperature, pressure, quasiparticles, phases of matter, classical trajectories, and macroscopic objects are real. They are not present as microscopic absolutes, but they are not illusions. They are stable quotient structures with predictive power, transformation laws, and domains of validity.

This matters for quantum-GR gluing because it prevents reduction in either direction. Quantum theory should not be forced to derive geometry from measurement alone. General relativity should not be treated as an eternal background immune to quantum revision. Both should be understood as quotient regimes. Quantum contextuality and geometric covariance are both real, both lawful, and both incomplete without a gluing theory.

Emergence also has levels. A classical particle trajectory may emerge from a quantum wavepacket under decoherence and approximation. A thermodynamic state may emerge from statistical ensembles. A smooth metric may emerge from deeper quantum-geometric degrees of freedom. A causal order may emerge from correlations, constraints, or algebraic relations. Each emergence is a quotient stabilization. Each requires specifying the policy, invariant, transition map, and domain of validity.

The final quantum-gravitational theory must therefore explain not only what replaces spacetime microscopically, but why classical spacetime emerges so reliably. It must explain why local inertial frames work, why causal order is stable, why clocks agree, why curvature behaves classically, why macroscopic records persist, and why quantum ambiguity is usually hidden from ordinary geometry. These are not peripheral questions. They are the mechanism by which quantum and GR descriptions glue in the world we actually observe.

Section 9 has reinterpreted decoherence and classical emergence as stabilization of quotient identities across broad families of admissibility policies. Decoherence is multi-policy stabilization. Classical spacetime is a robust quotient. Macroscopic objects are glued quotient families. Environmental interactions generate admissible agreement. Emergence does not reduce geometry to measurement alone; it identifies geometry as an objective invariant stabilized across lawful policies. This prepares the transition to holography, where nonlocal gluing between boundary and bulk descriptions suggests that spacetime identity may be even more radically quotient-based than classical geometry admits.

10. Holography, Boundary Reconstruction, and Nonlocal Gluing

Holography is one of the strongest indications that spacetime identity may be quotient-based rather than primitive. In holographic duality, a gravitational theory in a bulk spacetime may be equivalent to a nongravitational quantum theory associated with a lower-dimensional boundary. The two descriptions do not look the same. One speaks geometrically of bulk spacetime, horizons, causal structure, and gravitational dynamics. The other speaks algebraically of quantum degrees of freedom, states, operators, correlations, and entanglement. Yet the duality suggests that these different presentations may encode the same physical content under a deeper admissibility relation.

From the perspective of Admissibility Algebra, holography is not merely a surprising correspondence. It is a major example of admissible nonlocal gluing. A bulk gravitational presentation and a boundary quantum presentation may be distinct as presentations but equivalent as quotient objects under a holographic policy. The bulk and boundary are not naively identical. They are related by a transformation that preserves a hidden invariant structure. The question is therefore not whether the boundary literally is the bulk in the ordinary sense. The question is under what admissibility policy the boundary and bulk presentations may be treated as the same physical object.

This matters for quantum gravity because holography challenges the assumption that locality, dimension, and geometric containment are absolute. A region of bulk spacetime may be encoded nonlocally in boundary degrees of freedom. A geometric object may correspond to an

algebraic or entanglement structure. A black hole may be represented through thermal, quantum-informational, and geometric descriptions at once. These are precisely the kinds of cross-policy identifications that the quantum-GR gluing problem requires.

Holography therefore provides a working model, not necessarily the final model, of policy-indexed physical identity. It shows how a gravitational quotient and a quantum quotient may be different local charts of one deeper object. It also shows that the gluing map may be highly nonlocal, scale-sensitive, and dependent on boundary conditions. This section interprets holography as evidence for admissibility-covariant physics and examines how boundary reconstruction illuminates the gluing obstruction.

10.1 Holography as evidence for quotient equivalence

A holographic duality relates two descriptions that appear to have different primitive objects. On one side there may be a gravitational bulk with spacetime geometry. On the other side there may be a quantum boundary theory without gravity in the same ordinary sense. If the duality is exact, the two sides are not merely analogous. They are equivalent presentations of one physical structure. In admissibility language, they are distinct presentations X_{bulk} and X_{boundary} such that:

$$X_{\text{bulk}} \equiv \pi_{\text{holo}} X_{\text{boundary}}$$

where π_{holo} is the holographic admissibility policy.

This equivalence is not equality of form. The bulk and boundary may differ in dimension, locality structure, variable type, and geometric interpretation. Their equivalence lies in preserved physical content: spectra, correlations, thermodynamic quantities, entanglement structure, causal constraints, scattering data, or other invariant features appropriate to the duality. The holographic policy tells us which structures must match and which presentational differences are not physical.

This is the essence of quotient equivalence. A bulk metric and a boundary quantum state may look unrelated at the presentation level. Yet under a suitable policy, both may be quotient projections of a deeper invariant. The physical identity lies not in shared notation but in admissible transport between descriptions. If all physically meaningful quantities can be translated without loss, the presentations are equivalent under that policy.

Holography therefore supports the central thesis of this paper: physical identity is not exhausted by one presentation. A gravitational object may have a quantum-algebraic presentation. A quantum state may have a geometric presentation. A horizon may have thermodynamic, geometric, and informational quotient descriptions. A bulk region may be

encoded in boundary correlations. The same deeper object may become different physical objects under different admissibility policies.

This does not imply that every quantum theory is secretly a spacetime, or that every geometric theory has a simple boundary dual. Holographic equivalence is domain-specific and highly structured. Its existence is not a license for indiscriminate identification. It is evidence that in at least some regimes, quantum and gravitational identities can be glued through a nontrivial quotient relation. The admissibility policy is strict, not poetic.

The importance for quantum gravity is that holography avoids direct unification by direct sameness. It does not require the boundary variables to look like bulk metric variables. It does not require local bulk degrees of freedom to map to local boundary degrees of freedom in a naive way. It permits the same physical content to be represented through radically different identity structures. This is exactly what Admissibility Algebra predicts should be necessary when deep theories glue.

10.2 Bulk and boundary as incompatible presentations

Bulk and boundary descriptions are often incompatible at the level of presentation. The bulk description may use geometry, radial direction, horizons, geodesics, causal wedges, fields, and gravitational dynamics. The boundary description may use operators, states, correlation functions, entanglement, and nongravitational quantum dynamics. The dimensions may differ. The locality structures may differ. The variables may differ. The intuitive ontology may differ.

If one demands literal presentation equality, the duality is impossible. The bulk is not the boundary as a naive geometric object. A point in the bulk does not correspond simply to one point on the boundary. A local bulk field is not generally represented by a single local boundary operator. Bulk locality may emerge from highly organized boundary nonlocality. The radial direction may correspond to scale, entanglement, or renormalization-like structure rather than to an ordinary extra coordinate in the boundary theory.

This incompatibility is precisely what makes holography valuable. It demonstrates that physically equivalent descriptions need not share the same primitive admissibility policy. The bulk policy preserves geometric and diffeomorphic structure. The boundary policy preserves quantum algebraic structure. The holographic policy mediates between them. Without such mediation, the descriptions would appear mutually alien.

In Admissibility Algebra, bulk and boundary are different quotient presentations of a deeper physical object X :

$[X]\pi_{\text{bulk}}$ = bulk gravitational presentation

$[X]\pi_{\text{boundary}}$ = boundary quantum presentation

The holographic question is whether there exists a transition or equivalence:

$$T_{\text{bulk,boundary}}: [X]\pi_{\text{bulk}} \rightleftharpoons [X]\pi_{\text{boundary}}$$

that preserves the invariants demanded by both policies. When such a transition exists, the two incompatible presentations can be glued. When it does not, the apparent duality fails or remains approximate.

The incompatibility also protects against oversimplification. One should not say that the bulk is merely an illusion or that the boundary is merely a shadow. Each presentation is lawful under its policy. The bulk captures geometric relations naturally. The boundary captures quantum information and algebraic structure naturally. The deeper identity is not obtained by abolishing one side but by giving admissible transition data between them.

This structure resembles the quantum-GR problem itself. Quantum theory and general relativity are incompatible presentations if treated naively. Holography shows that incompatibility at the presentation level need not prevent equivalence at the quotient level. It suggests that the final quantum-GR glue may look less like direct equation-merger and more like a dual or higher-categorical equivalence between distinct admissibility charts.

10.3 Reconstruction as admissible transition

Boundary reconstruction is the attempt to recover bulk information from boundary data. In holographic settings, one asks how a bulk field, region, geometry, or operator can be represented in the boundary theory. From the admissibility viewpoint, reconstruction is an admissible transition map. It carries a quotient object expressed under boundary policy into a quotient object expressed under bulk policy, or conversely shows how a bulk object is encoded in boundary terms.

This transition cannot be arbitrary. It must preserve the relevant invariants. A reconstructed bulk operator must reproduce the correct boundary correlators. A reconstructed geometry must agree with entanglement, entropy, causal, or thermodynamic data encoded by the boundary. A reconstructed region must respect the appropriate accessibility conditions. If the reconstruction produces bulk features that cannot be distinguished or supported by boundary invariants, it has exceeded admissibility.

The reconstruction map therefore has the form:

$$R: [X]\pi_{\text{boundary}} \rightarrow [X]\pi_{\text{bulk}}$$

or, more carefully, a family of such maps defined only over suitable regions of policy space. In some regimes, reconstruction may be exact. In others, it may be approximate, perturbative,

code-subspace-relative, observer-relative, or obstructed. This limited domain is not a weakness of the concept. It is evidence that reconstruction is policy-indexed.

The same boundary data may support multiple bulk reconstructions depending on the admissibility policy. A coarse boundary policy may reconstruct only semiclassical geometry. A finer policy may encode quantum corrections. An entanglement-wedge policy may reconstruct one region but not another. A thermal or black-hole policy may reconstruct exterior geometry while encoding interior information in a more subtle way. These differences show that reconstruction is not a single universal inverse map. It is a structured family of admissible transitions.

This is especially important for locality. A local-looking bulk object may be represented nonlocally on the boundary. The reconstruction map may distribute one bulk degree of freedom across many boundary degrees of freedom. This means that locality is not necessarily preserved presentation-by-presentation. It is preserved, if at all, as an emergent invariant under the appropriate quotient. A local bulk field may be local under π_{bulk} and nonlocal under π_{boundary} , while both remain admissibly equivalent under π_{holo} .

Reconstruction also clarifies how gluing can be nonlocal without being lawless. Nonlocality in presentation does not imply arbitrary influence or violation of physical constraint. It means that the invariant content is encoded in a way that does not respect the same locality policy. The transition is lawful if it preserves causal, algebraic, entropic, and observational invariants. In Admissibility Algebra, nonlocal reconstruction is admissible when it preserves the identity that matters across policies.

Quantum gravity likely requires this kind of transition. The geometric locality of general relativity and the algebraic structure of quantum theory may not share one primitive local substrate. A quantum-gravitational theory may reconstruct locality only under certain policies, much as holography reconstructs bulk geometry from boundary data. The lesson is that gluing may be real even when locality, dimension, and object identity are not preserved in their naive forms.

10.4 Locality as a policy-relative phenomenon

Locality is one of the deepest assumptions in physics, but holography forces it to be refined. In general relativity, locality is organized by spacetime neighborhoods, causal cones, and metric relations. In quantum field theory, locality is reflected in commutation or anticommutation relations associated with spacelike separation, as well as in the assignment of algebras to spacetime regions. In holography, however, bulk locality may be encoded in boundary degrees of freedom in a way that is not boundary-local in the same sense.

This suggests that locality is policy-relative. A structure may be local under one policy and nonlocal under another. Bulk locality is local with respect to the emergent gravitational geometry. Boundary encoding may be local or nonlocal with respect to the boundary algebraic structure. The two locality notions need not coincide presentation-by-presentation. They must be related through admissible transition maps.

In notation:

$\text{local}\pi_{\text{bulk}}(X)$ need not imply $\text{local}\pi_{\text{boundary}}(X)$

and conversely. What matters is whether the transition between policies preserves the physical invariants associated with locality: causal accessibility, commutation structure, signal constraints, entanglement organization, and operational recoverability. Locality is not a single absolute predicate. It is a property of quotient presentation under a policy.

This does not make locality meaningless. It makes it more precise. Under a geometric policy, locality is defined by the metric and causal structure. Under an algebraic policy, locality is defined by subalgebras, commutation relations, and inclusion relations. Under a holographic policy, locality may be reconstructed from entanglement and boundary correlations. Under a quantum-gravitational policy, locality may be approximate, emergent, or obstructed.

The policy-relativity of locality helps resolve a major tension in quantum gravity. General relativity treats locality geometrically, but the geometry is dynamical. Quantum theory treats locality through algebra and causal background, but the background may not be fundamental. Holography suggests that locality may emerge from deeper nonlocal organization. Admissibility Algebra states the general rule: locality is the invariant that survives under policies capable of defining localized comparison. Where those policies fail to glue, locality fails to globalize.

This view also reframes black holes. The exterior, horizon, interior, and boundary descriptions may assign locality differently. An infalling observer, an exterior observer, and a boundary reconstruction may each use a different policy. The paradoxes arise when one tries to force these descriptions into one global locality policy. A gluing approach instead asks where the policies overlap, what invariants they share, and which attempted global localities are inadmissible.

The same applies to cosmology. A global spacetime description, a local observer's causal patch, and a quantum state of perturbations may not share one simple locality structure. Their compatibility depends on admissible transition. Locality becomes an emergent agreement among policies rather than an absolute given.

Thus holography teaches that quantum gravity should not assume locality as primitive in the classical sense. It should derive locality as a robust quotient where possible, identify nonlocal encodings where necessary, and mark obstruction where no global locality policy exists.

10.5 What holography teaches Admissibility Algebra

Holography teaches Admissibility Algebra five major lessons.

First, it teaches that radically different presentations may be physically equivalent. A geometric bulk and a quantum boundary can encode the same invariant content under a strict admissibility policy. This supports the broader claim that physical identity should be understood as policy-indexed quotient equivalence rather than literal sameness of presentation.

Second, it teaches that dimension may be a quotient feature. The bulk may have an emergent radial direction not present as an ordinary geometric dimension in the same way on the boundary. If dimension can change across admissible presentations, then dimensionality itself is not necessarily primitive. It is a structure preserved under some policies and transformed under others.

Third, it teaches that locality may be emergent and nonlocally encoded. Bulk locality may arise from boundary correlations, entanglement, or algebraic organization. Therefore locality should be treated as a policy-relative invariant, not an unquestioned absolute. This is essential for quantum gravity, where classical spacetime locality may fail.

Fourth, it teaches that reconstruction is a gluing map. Recovering bulk geometry from boundary data is not merely decoding information. It is transporting physical identity across admissibility policies. A reconstruction succeeds when it preserves the invariants required by both sides. It fails when the attempted transition identifies too much, too little, or the wrong structure.

Fifth, it teaches that equivalence can be exact in content while nontrivial in presentation. This is perhaps the deepest lesson. The same physical object may appear geometric in one chart and algebraic in another. It may appear local in one chart and nonlocal in another. It may appear gravitational in one chart and nongravitational in another. Admissibility Algebra supplies the rule for understanding this without collapsing into either relativism or naive realism: the equivalence is real only under a policy that specifies the preserved invariants and lawful transitions.

Holography also sets a standard for future quantum-GR proposals. It is not enough to claim that geometry emerges from information, or that information emerges from geometry. One must specify the admissibility policy, the quotient objects, the transition maps, the domain of validity, the preserved invariants, and the obstruction conditions. Holography is valuable because it gives

concrete examples of these requirements, even if it does not solve every regime of quantum gravity.

From the perspective of this paper, holography is an existence proof for the kind of thinking required. It shows that gravitational identity and quantum identity can sometimes be related through a nontrivial quotient equivalence. It also shows that the relation may be nonlocal, boundary-sensitive, code-like, and dependent on reconstruction. That is exactly the type of structure a sheaf or stack over policy space is meant to capture.

Section 10 has therefore interpreted holography as nonlocal gluing between quantum and gravitational quotient presentations. Bulk and boundary descriptions are incompatible as presentations but may be equivalent as quotient objects. Reconstruction acts as admissible transition. Locality becomes policy-relative. Holography teaches that quantum-GR gluing is possible only through explicit control of identity, invariants, and transition maps. The next section will state the Quantum-GR Gluing Principle directly and formulate the conditions under which quantum and geometric descriptions may become admissibly compatible.

11. The Quantum-GR Gluing Principle

The preceding sections have reframed quantum gravity as a problem of admissibility, quotient identity, and sheaf-theoretic gluing. Quantum theory and general relativity are not merely two incomplete languages awaiting mechanical merger. They are two mature quotient disciplines. Quantum theory defines physical identity through state spaces, operator algebras, measurement contexts, amplitudes, phase, entanglement, and admissible transformations. General relativity defines physical identity through diffeomorphism invariance, causal structure, metric relations, background independence, and geometric covariance. Both are powerful because both know how to discard false distinctions and preserve real invariants. Their conflict arises because their admissibility policies do not automatically agree.

The Quantum-GR Gluing Principle states that a successful theory of quantum gravity must provide a higher admissibility structure in which quantum and gravitational quotient presentations become compatible local descriptions of one deeper object. This deeper object need not resemble a classical spacetime, a Hilbert-space state, a metric operator, a path integral, a boundary algebra, or a spin network in any single primitive sense. Those may be charts, projections, or quotient presentations. The deeper theory is recognized by its ability to produce them under lawful policies and to explain when they glue.

This principle shifts the burden of theory construction. It is not enough to quantize the metric, geometrize the wavefunction, declare information fundamental, or identify spacetime with entanglement. Each of these may be useful, but each is incomplete unless it specifies the

admissibility policy under which its objects count as physical, the invariants it preserves, the quotient maps it imposes, and the transition data by which it relates to other regimes. The final theory must be admissibility-covariant.

The principle therefore does not compete directly with existing quantum gravity programs. It supplies a criterion above them. A candidate theory is not judged only by formal elegance or recovery of known limits, but by whether it solves the identity-gluing problem: whether it explains how quantum identity and gravitational identity are local shadows of one admissible structure.

11.1 Formal statement of the principle

Let Π be a policy space whose points π represent admissibility policies. These policies may include quantum measurement contexts, operator-algebraic regimes, gauge equivalence rules, diffeomorphic identity rules, observer frames, causal-access conditions, renormalization scales, coarse-graining maps, semiclassical limits, boundary reconstruction policies, and environmental stabilization regimes.

Let X denote a deeper physical presentation, not assumed to be fundamentally quantum or fundamentally geometric in the ordinary sense. For each admissibility policy $\pi \in \Pi$, the theory assigns a quotient presentation:

$$[X]_{\pi}$$

This quotient is the physical object X as made admissible under policy π . The invariant content preserved by π is:

$$\text{Inv}_{\pi}(X)$$

A quantum policy π_Q yields a quantum quotient:

$$[X]_{\pi_Q}$$

A gravitational policy π_{GR} yields a geometric quotient:

$$[X]_{\pi_{GR}}$$

The Quantum-GR Gluing Principle states:

Quantum theory and general relativity glue over a policy region $U \subset \Pi$ when their quotient presentations arise as compatible restrictions of a single admissibility-covariant object over U .

Equivalently, if U is covered by quantum, gravitational, semiclassical, observer-relative, boundary, and scale-dependent policy regions U_i , and if each region carries a quotient

presentation $s_i \in \mathcal{F}(U_i)$, then quantum-GR gluing occurs when the s_i agree admissibly on all overlaps and therefore descend from a common section $s \in \mathcal{F}(U)$.

In ordinary language:

Quantum gravity exists as a coherent theory when quantum and gravitational descriptions are not forced into literal equality, but are shown to be compatible quotient presentations of one deeper admissibility-indexed object.

The gluing condition is not presentation identity. A Hilbert-space description need not look like a Lorentzian metric. A boundary algebra need not look like a bulk geometry. A path integral need not look like a canonical state. A spin-network description need not look like a smooth manifold. What must agree is the invariant content on admissible overlaps.

Thus, for an overlap policy π_{QG} , one requires:

$$\text{Inv}\pi_{\text{QG}}([X]\pi_{\text{Q}}) = \text{Inv}\pi_{\text{QG}}([X]\pi_{\text{GR}})$$

where the equality means admissible equivalence under the overlap policy, not literal equality of notation. If the quantum quotient and the gravitational quotient preserve the same overlap invariants and their transformations cohere, then they glue. If not, the failure defines an obstruction.

The principle may be stated more compactly:

A physical theory is quantum-gravitational only if it supplies quotient presentations that recover quantum theory and general relativity as local admissibility charts and provides coherent transition data between those charts.

This statement is deliberately structural. It does not prescribe one primitive ontology. It prescribes what any primitive ontology must accomplish.

11.2 Conditions for admissible quantum-geometric compatibility

Quantum-geometric compatibility requires more than agreement on a few limiting equations. It requires preservation of the identity structures that make quantum theory and general relativity successful. At minimum, the overlap policy must preserve quantum probability, algebraic observability, entanglement-relevant correlations, causal-geometric structure, diffeomorphic admissibility, and appropriate conservation or constraint relations.

The first condition is invariant preservation. A proposed transition between quantum and geometric descriptions must preserve quantities that both sides can legitimately compare: energy, momentum, entropy, causal accessibility, correlation structure, boundary data, scattering information, curvature in an appropriate limit, and observational probabilities. If a

transition preserves geometry but destroys quantum correlations required by the quantum policy, it is inadmissible. If it preserves quantum amplitudes but depends on coordinate artifacts forbidden by gravitational admissibility, it is inadmissible.

The second condition is equivalence respect. If two quantum presentations are equivalent under a quantum policy, their geometric images must be equivalent under the corresponding geometric or overlap policy. Likewise, if two geometric presentations are diffeomorphically equivalent, their quantum images must not differ in physical content. A transition map that sends gauge redundancy or coordinate redundancy into observable difference has failed to respect admissibility.

The third condition is compositional coherence. Physical transformations must compose lawfully across regimes. If one moves from quantum to semiclassical to geometric description, the result must agree, where defined, with a direct quantum-to-geometric transition. If one changes observer frame, then changes scale, then reconstructs geometry, the result must be compatible with changing scale first and observer frame second, whenever both paths are admissible. Failure of such coherence indicates a higher obstruction.

The fourth condition is locality discipline. The theory must specify when locality is geometric, when it is algebraic, when it is emergent, when it is reconstructed, and when it fails. A quantum-geometric transition may be nonlocal in one presentation while local in another, but this cannot be uncontrolled. It must preserve the physically relevant causal and operational invariants under the policy in force.

The fifth condition is temporal discipline. The theory must explain how external time, proper time, relational time, thermal time, emergent time, or timeless constraint structure arise under different policies. A candidate theory that assumes external time without recovering background independence has not glued GR. A candidate theory that erases dynamical time without recovering quantum transition probabilities has not glued quantum theory.

The sixth condition is measurement compatibility. Quantum measurement policies must be embedded in gravitationally admissible structure. A measurement must be localizable or relationally specified without relying on forbidden background labels. Records must be describable as physical quotient structures inside the universe, not as external primitives. Apparatus, observer, outcome, and causal access must be admissible under both quantum and gravitational constraints.

The seventh condition is classical recovery. The theory must recover ordinary quantum mechanics in appropriate nongravitational or weak-gravity regimes and recover general relativity in appropriate classical macroscopic regimes. More strongly, it must explain why these

recoveries are stable. It must show why the quantum quotient and the geometric quotient become robust where they do.

The eighth condition is obstruction accounting. Where gluing fails, the theory must say why. It must identify whether the failure concerns time, locality, observability, Hilbert-space factorization, diffeomorphism invariance, horizon information, singularity structure, or policy incompatibility. A theory that hides its obstructions is incomplete. A theory that classifies them has begun to explain quantum gravity.

These conditions define admissible quantum-geometric compatibility. They are demanding because the problem is demanding. Quantum gravity is not the art of making two vocabularies resemble each other. It is the construction of a transition system that preserves the lawful identities of both.

11.3 Classical gravity as coarse-policy quotient

Classical gravity appears when the deeper physical object X is viewed under a coarse geometric admissibility policy. This policy suppresses quantum-gravitational distinctions that are irrelevant at macroscopic scales and preserves metric, causal, curvature, and stress-energy relations. The result is the classical gravitational quotient:

$[X]\pi_{\text{classical_GR}}$

This quotient is not false because it is coarse. It is physically real within its domain because it is stable, predictive, and widely glued across policies. It is recovered by clocks, rods, light propagation, orbital motion, curvature measurements, gravitational waves, and causal structure. It is also mathematically disciplined by diffeomorphism invariance and covariance. Classical GR is therefore a highly successful quotient presentation.

In this view, the Einstein field equations describe lawful relations inside a stabilized quotient regime. They are not necessarily equations of the deepest presentation X . They are equations governing $[X]\pi_{\text{classical_GR}}$. This distinction matters. If the quotient fails near singularities, horizons, or Planckian curvature, the failure does not mean that reality fails. It means the classical gravitational admissibility policy no longer produces a valid smooth quotient.

The smooth manifold, metric tensor, and causal structure should therefore be treated as emergent but robust. They are not arbitrary conventions. They are invariant structures surviving a wide range of admissibility policies. Their robustness explains why general relativity works so well. Their quotient status explains why they may break down in quantum-gravitational regimes.

A coarse-policy quotient also explains why many microscopic distinctions can be ignored gravitationally. The classical metric responds to coarse stress-energy, not to every inaccessible phase relation in a simple way. A planet's gravitational field is stable despite quantum

complexity within its matter. A macroscopic spacetime region is described geometrically despite underlying molecular, nuclear, quantum, and thermodynamic degrees of freedom. The geometric policy preserves the relational structure needed for gravity and discards the rest.

However, the coarse quotient cannot be absolute. Black holes, early-universe cosmology, and high-curvature regimes reveal that quantum distinctions may become gravitationally relevant. Entropy, information, horizon structure, vacuum fluctuations, and quantum stress-energy cannot always be ignored. In those regimes, the policy $\pi_{\text{classical_GR}}$ becomes insufficient. It must be refined or replaced by a quantum-geometric policy.

Classical gravity is therefore not the enemy of quantum gravity. It is a target quotient. A successful theory must produce it, explain its stability, and identify its boundary of validity. The deeper theory must say why classical geometry glues across ordinary policies and why that gluing fails in extreme regimes.

11.4 Quantum theory as fine-policy contextual quotient

Quantum theory appears when the deeper physical object X is viewed under policies that preserve phase, superposition, entanglement, operator structure, contextual measurement relations, and probabilistic amplitudes. These policies are finer than classical policies because they preserve distinctions that classical coarse-graining discards. The resulting quotient is:

$[X]\pi_{\text{quantum}}$

This quotient is not simply more detailed in every respect. It preserves quantum distinctions while often depending on structures such as measurement context, observable algebra, subsystem decomposition, or background causal order. It is fine with respect to phase and entanglement, but it may still quotient away other information, such as environmental degrees of freedom or global geometric structure. Thus quantum theory is not an absolute unquotiented view. It is a specific family of fine contextual quotients.

Quantum contextuality arises because different fine policies do not always glue into a single classical presentation. One observable context preserves distinctions that another cannot jointly preserve. A measurement basis selects one quotient. An incompatible measurement basis selects another. The quantum object is the structured family of these policy-relative quotients, governed by probability rules and algebraic constraints.

This makes quantum theory an ideal prototype for admissibility-covariant physics. It already teaches that a physical object may not have one global policy-independent identity. It teaches that measurement is quotient selection, that noncommuting observables obstruct naive globalization, and that entanglement makes subsystem identity relational. Quantum gravity must preserve these lessons.

However, ordinary quantum theory often assumes background structures that gravity problematizes. It may rely on a fixed time parameter, a fixed causal order, a fixed Hilbert-space factorization, or a fixed spacetime background. These assumptions are admissible in many regimes but not universally. Therefore the quantum quotient must itself be made gravitationally admissible.

A quantum-gravitational policy π_{QG} must preserve the essential fine quantum distinctions without assuming forbidden background structure. It must allow amplitudes, probabilities, entanglement, and observables to be defined relationally, covariantly, or emergently where necessary. This is a much harder task than simply applying quantization rules to gravitational variables.

Quantum theory as fine-policy contextual quotient also explains why quantum behavior usually disappears from classical gravity. The fine distinctions preserved by π_{quantum} may not survive coarse geometric policies. Decoherence, environmental stabilization, thermodynamic averaging, and scale separation suppress or hide them. But in black holes, cosmology, and high-curvature regimes, those distinctions may re-enter the geometric quotient. That is where quantum gravity becomes unavoidable.

Thus quantum theory and classical gravity are not two rival absolutes. They are different quotient projections of physical reality. The quantum quotient is fine, contextual, and algebraic. The gravitational quotient is coarse, geometric, and diffeomorphic. Quantum gravity must identify the higher structure in which both are admissible.

11.5 Quantum gravity as a stack of partially glued descriptions

The most natural mathematical image of quantum gravity in this framework is a stack of partially glued descriptions over policy space Π . Each region of Π supports a category of admissible quotient presentations. Some regions look quantum. Some look geometric. Some look semiclassical. Some look thermodynamic. Some look holographic. Some look algebraic. Some look observer-relative. The full theory is not one flat object, but a structured descent system among these local regimes.

In such a stack, objects are quotient presentations. Morphisms are admissible transformations. Higher morphisms are equivalences between transformations. Restriction maps carry descriptions to narrower policy regions. Transition maps relate overlapping regimes. Descent data specify when local descriptions glue. Obstruction data specify when they do not.

Quantum gravity then becomes neither a single classical spacetime nor a single ordinary quantum state. It becomes a higher admissibility object whose local shadows include both. In low-energy geometric regions, the stack descends to classical general relativity. In fine quantum regions, it descends to quantum theory. In semiclassical regions, it produces approximate

transitions. In holographic regions, it permits nonlocal boundary-bulk equivalence. In obstructed regions, it marks the failure of classical spacetime, ordinary locality, external time, or naive observable identity.

This view also explains why no existing approach has fully ended the problem. Each may capture a legitimate chart. Perturbative quantum gravity captures one chart. Quantum field theory in curved spacetime captures another. Canonical approaches capture another. Loop variables capture another. Path integrals capture another. Causal structures capture another. Holography captures another. Effective field theory captures another. Algebraic approaches capture another. The missing object may not be any one chart, but the admissibility stack that explains their relations.

Partial gluing is important. Physics often advances by partial glues before complete unification. Semiclassical gravity is a partial glue. Effective quantum gravity is a partial glue. Holography is a deep but domain-specific glue. Decoherence is a glue between quantum and classical descriptions. Thermodynamic gravity proposals are glues between geometry and entropy. Each one identifies an overlap region in Π . The question is how these overlaps compose.

A stack also permits genuine obstruction. It does not require that all descriptions glue globally. Some may not. There may be no global time policy. There may be no global classical spacetime. There may be no global Hilbert-space factorization. There may be no observer-independent interior-exterior description of a black hole in the naive sense. These failures need not be fatal. They may be structural facts about the stack.

The Quantum-GR Gluing Principle therefore culminates in this formulation:

Quantum gravity is the admissibility-covariant stack whose local quotient presentations reproduce quantum theory, general relativity, semiclassical physics, holographic reconstruction, and classical emergence, while whose obstruction structure explains the failure of absolute time, absolute locality, absolute geometry, and policy-independent physical identity.

This is the central program of the paper. It does not claim that the final equations are already known. It claims that the final equations must live inside a higher identity structure. Without that structure, equations may be formally impressive but physically underdetermined. With that structure, quantum gravity can be understood as lawful gluing rather than forced unification.

Section 11 has stated the Quantum-GR Gluing Principle, identified the conditions for admissible compatibility, interpreted classical gravity as a coarse-policy quotient, interpreted quantum theory as a fine-policy contextual quotient, and proposed quantum gravity as a stack of partially glued descriptions. The final section draws out the consequences for mathematical physics and explains why the future of quantum gravity may depend less on finding a single object and more on formalizing admissible identity itself.

12. Consequences for Mathematical Physics

The admissibility-covariant formulation changes the conceptual center of mathematical physics. It does not replace equations, experiments, geometry, algebra, or quantization. It asks a prior question: under what identity policy do those structures count as physical? A theory is not complete merely because it has variables and dynamics. It must also say which presentations are equivalent, which transformations preserve physical content, which invariants survive change of context, and how local descriptions glue into broader objects. Quantum gravity exposes this requirement because quantum theory and general relativity use different identity disciplines.

This perspective turns quantum gravity from a search for one master presentation into a search for a higher compatibility structure. The desired theory may not be best understood as a single equation written on a single background, a single Hilbert space, a single configuration space, or a single geometric object. It may be a policy-indexed system whose local quotient presentations appear as quantum mechanics, quantum field theory, general relativity, thermodynamics, holography, and semiclassical physics under different admissibility regimes. The deeper object is recognized not by its resemblance to one familiar formalism, but by its ability to generate and relate many such formalisms lawfully.

The consequences are substantial. Absolute physical identity gives way to admissibility-indexed identity. Spacetime becomes a stabilized quotient rather than an unquestioned primitive. Quantum contextuality becomes a local failure of classical gluing rather than a mere interpretive puzzle. Decoherence becomes multi-policy stabilization. Holography becomes nonlocal quotient equivalence. Gauge symmetry and diffeomorphism invariance become special cases of admissible identity. The problem of time becomes a failure of temporal policy globalization. Black hole puzzles become conflicts among observer, horizon, quantum, and geometric admissibility policies.

Mathematical physics should therefore expand its foundational vocabulary. Alongside state, observable, field, equation, symmetry, and geometry, it needs presentation, policy, quotient, invariant, admissible transition, gluing, obstruction, and policy covariance. These are not philosophical ornaments. They are the identity mechanics underlying the successful use of modern physical theories.

12.1 Reframing the search for quantum gravity

The traditional search for quantum gravity often begins with a choice of primitive. One program begins with strings. Another begins with loops or spin networks. Another begins with causal sets. Another begins with amplitudes or histories. Another begins with algebras. Another begins with entanglement. Another begins with effective field theory. Another begins with holographic

boundary data. Each may capture genuine structure. But from the admissibility viewpoint, the more important question is not simply what the primitive is. The question is what admissibility policy makes the primitive physical.

A primitive object without an identity discipline is not yet a theory. A metric component is not physical unless diffeomorphism redundancy is handled. A quantum state is not physical unless phase, measurement context, and observable structure are specified. A causal relation is not physical unless one knows what transformations preserve it. A boundary degree of freedom is not physical unless one knows what bulk or boundary invariants it encodes. A spin network, string state, path integral, or algebraic net becomes physical only under a policy that identifies its admissible transformations and quotient content.

This reframes comparison among quantum gravity programs. Instead of asking only which program is correct, one may ask which policy region each program covers. Effective quantum gravity covers a perturbative low-energy region. Quantum field theory in curved spacetime covers a regime where matter is quantum but geometry remains classical. Holography covers regimes where boundary quantum descriptions and bulk gravitational descriptions are related by a nonlocal equivalence. Canonical approaches emphasize constraints and background independence. Algebraic approaches emphasize observables and locality. Each may be a chart in a larger admissibility atlas.

The failure of a program to cover every regime need not make it worthless. It may indicate that the program is a valid local section rather than the global object. Conversely, success in one region does not justify elevating one chart into an absolute ontology. The final theory may require descent data among many charts. Its unity may be gluing unity, not presentation unity.

This also changes the meaning of progress. A new quantum gravity result is valuable not only when it proposes a new fundamental object, but when it clarifies a transition map, identifies an obstruction, proves compatibility of quotient presentations, derives a stable classical limit, reconstructs locality, or explains why a policy fails. Mapping the admissibility atlas is itself progress.

The central research question becomes:

Which admissibility policies recover known physics, and what higher structure relates them?

This question is more flexible than traditional unification and more rigorous than informal pluralism. It allows many local descriptions while demanding precise gluing laws. It accepts that quantum gravity may not have one humanly simple presentation, while insisting that the relations among presentations must be lawful.

12.2 The end of absolute physical identity

Classical intuition treats physical identity as absolute. An object is what it is. A point is where it is. A property has the value it has. A trajectory follows the path it follows. Modern physics has already weakened each of these assumptions. Quantum theory denies simultaneous classical value assignment for incompatible observables. General relativity denies absolute coordinate identity. Gauge theory denies physical uniqueness of gauge presentations. Thermodynamics denies relevance of microscopic detail for macroscopic identity. Holography denies that dimension and locality must be presentation-invariant in the naive sense.

Admissibility Algebra makes the general lesson explicit: physical identity is not absolute equality of presentations. It is lawful equivalence under a policy. To say that two descriptions are the same physical object is to say that their differences are inadmissible as physical differences under the relevant identity rule. The rule may be quantum, geometric, gauge-theoretic, thermodynamic, holographic, operational, or scale-relative. What matters is that it is explicit and constrained.

This does not dissolve reality. It disciplines realism. A policy-indexed object is not a subjective invention. It is a quotient whose invariants survive lawful transformation. The electromagnetic field is real because gauge-invariant content survives gauge changes. Spacetime geometry is real because diffeomorphism-invariant relations survive coordinate changes. Quantum probabilities are real because they survive admissible preparation and measurement transformations. Temperature is real because it survives microscopic fluctuations under thermodynamic policy.

Absolute identity is replaced by invariant survival. The more broadly a quotient survives across admissibility policies, the more classical and objective it appears. The less broadly it survives, the more contextual, quantum, emergent, or regime-dependent it appears. Objectivity becomes broad admissible gluing, not freedom from all policy.

This shift is especially important for quantum gravity. If one demands absolute identity, one will ask for a single global time, a single global geometry, a single global Hilbert-space factorization, a single observer-independent black hole interior description, and a single locality policy. These demands may be too strong. The correct theory may instead contain many local identities, some of which glue and some of which do not. Failed globalization would then be a structural fact rather than a paradox.

The end of absolute physical identity also changes the philosophical interpretation of physics. The world is not reduced to descriptions, but neither is it accessible apart from admissible equivalence. Reality appears through what lawful transformations cannot destroy. A physical object is the stable residue of admissibility, not the raw notation used to present it.

12.3 Admissibility-covariant laws

A law of physics is usually thought of as an equation invariant under certain transformations. Admissibility covariance generalizes this idea. A law should not merely be invariant under coordinate change, gauge transformation, or unitary evolution. It should specify how its physical content transports across admissibility policies. A law becomes stronger when it survives not only changes of presentation inside one theory, but changes of policy between theories, scales, observers, and regimes.

Ordinary covariance is the model. General relativity demands that laws be expressible independently of arbitrary coordinate choice. Gauge theory demands that observable content be independent of gauge presentation. Quantum theory demands that probabilities and amplitudes transform consistently under admissible unitary change. Renormalization demands that effective laws transform coherently across scale. Admissibility covariance asks for the common structure behind all of these.

An admissibility-covariant law has four components. First, it identifies the presentations to which it applies. Second, it identifies the policy under which those presentations become physical. Third, it identifies the invariants preserved by the law. Fourth, it identifies how the law transforms under admissible policy change. Without the fourth component, the law may be valid locally but disconnected from neighboring regimes.

For quantum gravity, the needed laws must transport across quantum and geometric policies. They must explain how quantum amplitudes relate to causal structure, how entanglement relates to geometry, how metric relations emerge from deeper degrees of freedom, how measurement records become gravitationally admissible, how energy and information are preserved or transformed near horizons, and how classical spacetime becomes stable. These are not merely dynamical questions. They are covariance questions at the level of admissibility.

A law that works only in one chart may be a valid local law. A law that transports across charts is a deeper law. The final quantum-gravitational law may therefore look less like a single familiar differential equation and more like a rule of descent, reconstruction, obstruction, or invariant transport. Its mathematical form may be categorical, sheaf-theoretic, type-theoretic, algebraic, geometric, or hybrid.

Admissibility-covariant laws also clarify approximation. An approximate law is not simply a false law. It is a law valid under a restricted policy. Newtonian gravity is valid under a weak-field, low-velocity, classical policy. Quantum mechanics without gravity is valid under a policy where spacetime is treated as fixed or irrelevant. Quantum field theory in curved spacetime is valid under a policy where matter is quantum and geometry is classical. Effective field theory is valid under a scale policy. Each law has a domain in Π .

The final theory should therefore include a map of law domains. It should say where each familiar law is a quotient law, what policy produces it, how it transitions to neighboring laws, and where obstruction prevents extension. This is a more exact form of correspondence principle: not merely that old laws are recovered in limits, but that old laws are quotient sections over identifiable admissibility regions.

12.4 Testable and formal research directions

The admissibility-covariant program suggests several research directions. Some are formal, some conceptual, and some potentially physical. The immediate task is not to claim experimental predictions prematurely, but to create a mathematical framework precise enough to distinguish genuine gluing from analogy.

The first direction is formal construction of policy spaces. One must define Π for restricted physical domains. A simple starting point would include measurement contexts, gauge policies, diffeomorphism policies, scale policies, and observer-access policies. The aim would be to identify refinement relations, overlap regions, incompatible policies, and transition paths. Even a finite or toy-free restricted model, built from real physical structures rather than artificial examples, could clarify the architecture.

The second direction is construction of admissibility sheaves or stacks for known theories. General relativity can be represented as a quotient geometry under diffeomorphism policies. Quantum mechanics can be represented as contextual quotient data over measurement policies. Gauge theory can be represented through quotient categories of connections and observables. Thermodynamics can be represented as coarse-grained quotient structure over scale and macrovariable policies. These should then be compared as local cases of one general admissibility formalism.

The third direction is obstruction classification. Known foundational problems should be rewritten as failed gluing problems. The problem of time becomes failure of temporal policy globalization. Quantum contextuality becomes failure of global value-section gluing. Black hole information puzzles become conflict among horizon, exterior, interior, unitary, and gravitational policies. Singularities become failure of smooth geometric quotient extension. Anomalies become failure of symmetry or admissibility preservation under quantization.

The fourth direction is functorial comparison of quantum and geometric regimes. Existing partial bridges should be treated as functors or transition maps: semiclassical gravity, quantum field theory in curved spacetime, effective quantum gravity, holographic reconstruction, algebraic nets, entanglement-geometry correspondences, and thermodynamic gravity relations. Each should be evaluated by preserved invariants, domain of validity, naturality, and obstruction.

The fifth direction is classical emergence as quotient stabilization. Decoherence, quantum Darwinism, thermodynamic irreversibility, and macroscopic record formation should be unified as multi-policy stabilization. The research question is: which quotient structures become robust across many environmental policies, and how does this produce the apparent objectivity of classical spacetime and measurement records?

The sixth direction is relational observables and measurement in quantum gravity. A measurement theory compatible with diffeomorphism invariance must define apparatus, record, location, and outcome relationally. This is where quantum theory and GR most urgently require gluing. One should formalize measurement policies that preserve quantum probabilities while avoiding forbidden background labels.

The seventh direction is holographic admissibility. Holography should be analyzed as a nonlocal quotient equivalence. The formal question is: what is the policy under which bulk and boundary descriptions are identical? Which invariants define the equivalence? Which reconstruction maps are admissible? Where do they fail? How does locality change across the transition?

The eighth direction is computational representation. Policy-indexed quotient structures may be expressible as typed graphs, categories, proof systems, or symbolic transition networks. Such representations could help map known theories, identify missing transition maps, and test whether proposed gluing diagrams commute. The goal is not numerical simulation alone, but structural verification of admissibility.

The ninth direction is possible phenomenology. If admissibility gluing fails in extreme regimes, there may be signatures in black hole evaporation, horizon entropy, early-universe perturbations, quantum reference frames, gravitational decoherence, causal-order experiments, or semiclassical backreaction. Such work must be cautious. The present framework is foundational first. But a good foundational theory should eventually identify where quotient policies fail observably.

The tenth direction is formal comparison with HoTT and univalent foundations. Policy-indexed identity should be represented as a type-theoretic or higher-categorical structure. The aim would be to state what it means for quantum and gravitational equivalences to be equivalent, for properties to transport across such equivalences, and for obstruction to appear as failure of higher coherence.

Together these directions turn the paper's thesis into a program. The program is not to declare quantum gravity solved. It is to specify the identity architecture any solution must satisfy.

12.5 Final synthesis: reality as lawful gluing

The central claim of this paper is that quantum theory and general relativity do not primarily fail to unify because their equations are incompatible on the surface. They fail because their admissibility policies for physical identity do not automatically glue. Quantum theory defines sameness through contextual algebra, state structure, phase, entanglement, measurement, and unitary or operational equivalence. General relativity defines sameness through diffeomorphism invariance, causal geometry, covariance, and background independence. These are both profound identity disciplines. Quantum gravity must respect both.

The proposed solution is not to choose one and erase the other. It is to construct a higher admissibility-covariant structure in which both appear as local quotient presentations. A physical presentation X becomes $[X]\pi$ under policy π . Quantum mechanics, classical gravity, semiclassical theory, holography, thermodynamics, and macroscopic classicality are different quotient regimes. Their relations are transition maps. Their compatibility is gluing. Their failures are obstructions. Their objectivity is invariant survival across policy space.

This gives a new interpretation of reality in mathematical physics. Reality is not raw presentation. It is not arbitrary perspective. It is the lawful gluing of admissible quotient presentations. What exists robustly is what survives many legitimate transformations. What exists contextually is what survives only within restricted policies. What appears paradoxical is often a forced globalization of identities that do not glue.

Quantum mechanics already taught that not all observables globalize into one classical value assignment. General relativity already taught that coordinate identity is not physical identity. Gauge theory already taught that many presentations can describe one field. Thermodynamics already taught that coarse quotients can be real. Holography already taught that geometry and quantum information may be radically different presentations of one content. Admissibility Algebra gathers these lessons into one principle.

The quantum-GR gluing obstruction is therefore not a peripheral problem. It may be the central clue. The obstruction says that ordinary physical identity is too weak, too flat, or too absolute for the deepest theory. The next layer of mathematical physics must formalize identity itself: presentations, policies, quotients, invariants, transitions, gluing, and obstruction.

The final slogan can now be stated precisely:

Quantum theory and general relativity do not disagree about reality; they disagree about admissible identity.

If that is correct, then quantum gravity will not be merely the quantization of geometry or the geometrization of quantum theory. It will be the admissibility-covariant theory of lawful gluing,

where spacetime, quantum state, measurement, locality, time, and objecthood appear as stable quotient shadows of a deeper identity structure.

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